

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A - \lambda I = \begin{vmatrix} a - \lambda & b \\ -b & a - \lambda \end{vmatrix} = (a - \lambda)^2 + b^2 = \lambda^2 - 2a\lambda + a^2 + b^2$$

$$\text{Thus } \lambda = \frac{2a \pm \sqrt{4a^2 - 4(a^2 + b^2)}}{2} = \frac{2a \pm \sqrt{-4b^2}}{2} = a \pm bi$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ implies } \begin{cases} x_1' = ax_1 + bx_2 \\ x_2' = -bx_1 + ax_2 \end{cases}$$

Change to polar coordinates:  $r^2 = x_1^2 + x_2^2$  and  $\tan\theta = \frac{x_2}{x_1}$

Take derivative with respect to  $t$  of both equations:

$$2rr' = 2x_1x_1' + 2x_2x_2' \text{ implies}$$

$$rr' = x_1(ax_1 + bx_2) + x_2(-bx_1 + ax_2)$$

$$= ax_1^2 + bx_1x_2 - bx_1x_2 + ax_2^2 = a(x_1^2 + x_2^2) = ar^2$$

Thus  $rr' = ar^2$  implies  $\frac{dr}{dt} = ar$  and thus  $r = Ce^{at}$ .

$$(\sec^2\theta)\theta' = \frac{x_1x_2' - x_1'x_2}{x_1^2} = \frac{x_1(-bx_1 + ax_2) - (ax_1 + bx_2)x_2}{x_1^2}$$

$$= \frac{-bx_1^2 + ax_1x_2 - ax_1x_2 - bx_2^2}{x_1^2} = \frac{-b(x_1^2 + x_2^2)}{x_1^2} = \frac{-b(r^2)}{x_1^2} = -b\sec^2\theta$$

$(\sec^2\theta)\theta' = -b\sec^2\theta$  implies  $\theta' = -b$  and thus  $\theta = -bt + \theta_0$