Section 7.5 (where A has 2 real distinct nonzero eigenvalues): Solve $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix} \mathbf{x}$

Step 1: Find eigenvalues:

$det(A - rI) = \begin{bmatrix} -2-r & 0 \\ 21 & 5-r \end{bmatrix} =$

Step 2: For each eigenvalue, find one eigenvector. I.e., find one NONZERO solution to $(A - rI)\mathbf{v} = \mathbf{0}$

General solution: ______________________________

Initial Value: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, IVP solution: ______________________

For this IVP solution:

$x_1(t) =$ ______________________________

$x_2(t) =$ ______________________________

Solve for x_1 in terms of x_2:

Answer the following questions for $A = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix}$:

The smaller eigenvalue of A is $r_1 =$ ____. An eigenvector corresponding to r_1 is $\mathbf{v} =$

The larger eigenvalue of A is $r_2 =$ ____. An eigenvector corresponding to r_2 is $\mathbf{w} =$

The general solution to $\mathbf{x}' = A\mathbf{x}$ is

For large **positive** values of t which is larger: $e^{r_1 t}$ or $e^{r_2 t}$?

For the following problems, consider the case when $c_1 \neq 0$ and $c_2 \neq 0$ where the general solution is $\mathbf{x} = c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t} + c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$,

For large **positive** values of t, which term dominates: $c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t}$ or $c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$?

Thus for large **positive** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

* moves away from the origin.
* moves toward the origin.
* approaches the line $y = mx$ with slope $m =$ ______________
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ______________. Note this case corresponds to where both $||c_1\mathbf{v}||e^{r_1 t}$ and $||c_2\mathbf{w}||e^{r_2 t}$ are large, but one is significantly larger than the other.

For large **negative** values of t which is larger: $e^{r_1 t}$ or $e^{r_2 t}$?

For large **negative** values of t, which term dominates: $c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t}$ or $c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$?

Thus for large **negative** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

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* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ______________. Note this case corresponds to where both $||c_1\mathbf{v}||e^{r_1 t}$ and $||c_2\mathbf{w}||e^{r_2 t}$ are large, but one is significantly larger than the other.

Give that the solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ in the

t, x_1-plane $\qquad$ t, x_2-plane $\qquad$ x_1, x_2-plane

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the

t, x_1-plane $\qquad$ t, x_2-plane $\qquad$ x_1, x_2-plane

The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$.

$\frac{dx_2}{dx_1} =$ ________________

Plot several direction vectors where
the slope is 0 and where slope is vertical.

Graph several trajectories.

Section 7.5 (where A has 2 real distinct nonzero eigenvalues): Solve $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix} \mathbf{x}$

Step 1: Find eigenvalues:

$$det(A - rI) = \begin{bmatrix} -2-r & 0 \\ -9 & -5-r \end{bmatrix} =$$

Step 2: For each eigenvalue, find one eigenvector. I.e., find one NONZERO solution to $(A - rI)\mathbf{v} = \mathbf{0}$

General solution: ____

Initial Value: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, IVP solution: ________________________

For this IVP solution:

$x_1(t) =$ ____

$x_2(t) =$ ____

Solve for x_1 in terms of x_2:

Answer the following questions for $A = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix}$:

The smaller eigenvalue of A is $r_1 =$ ____. An eigenvector corresponding to r_1 is $\mathbf{v} =$

The larger eigenvalue of A is $r_2 =$ ____. An eigenvector corresponding to r_2 is $\mathbf{w} =$

The general solution to $\mathbf{x}' = A\mathbf{x}$ is

For large **positive** values of t which is larger: $e^{r_1 t}$ or $e^{r_2 t}$?

For the following problems, consider the case when $c_1 \neq 0$ and $c_2 \neq 0$ where the general solution is $\mathbf{x} = c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t} + c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$,

For large **positive** values of t, which term dominates: $c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t}$ or $c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$?

Thus for large **positive** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

* moves away from the origin.
* moves toward the origin.
* approaches the line $y = mx$ with slope $m =$ ______________
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ______________. Note this case corresponds to where both $||c_1\mathbf{v}||e^{r_1 t}$ and $||c_2\mathbf{w}||e^{r_2 t}$ are large, but one is significantly larger than the other.

For large **negative** values of t which is larger: $e^{r_1 t}$ or $e^{r_2 t}$?

For large **negative** values of t, which term dominates: $c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t}$ or $c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$?

Thus for large **negative** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

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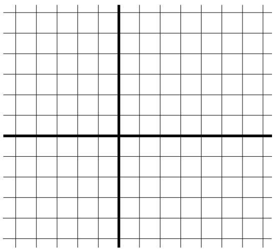
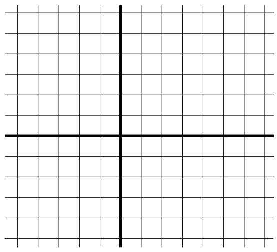
Give that the solution to $\mathbf{x}' = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{-2t}$

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ in the

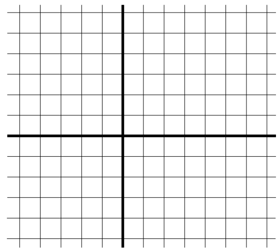
t, x_1-plane



t, x_2-plane

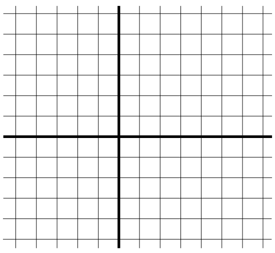


x_1, x_2-plane

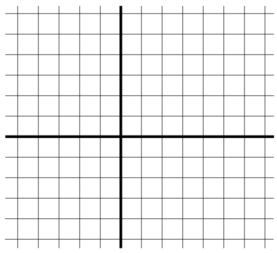


Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the

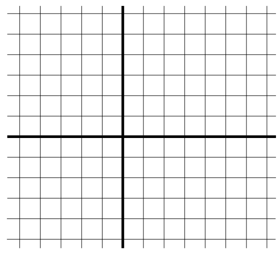
t, x_1-plane



t, x_2-plane



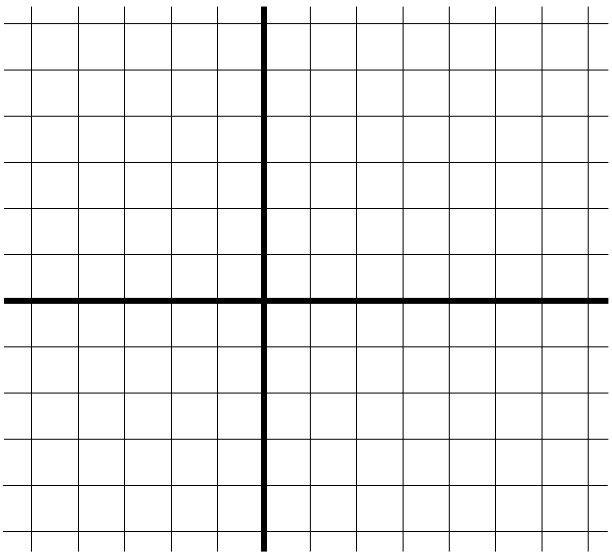
x_1, x_2-plane



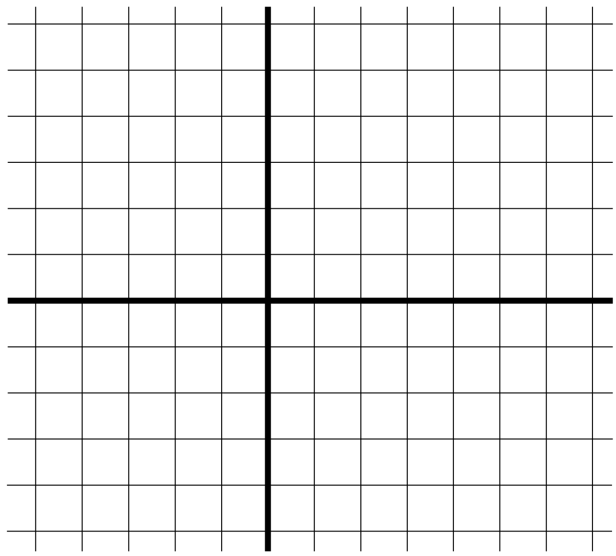
The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$.

$\frac{dx_2}{dx_1} =$ ________________

Plot several direction vectors where the slope is 0 and where slope is vertical.



Graph several trajectories.



Section 7.5 (where A has 2 real distinct nonzero eigenvalues): Solve $\mathbf{x}' = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix} \mathbf{x}$

Step 1: Find eigenvalues:

$$det(A - rI) = \begin{bmatrix} 2-r & 0 \\ 9 & 5-r \end{bmatrix} =$$

Step 2: For each eigenvalue, find one eigenvector. I.e., find one NONZERO solution to $(A - rI)\mathbf{v} = \mathbf{0}$

General solution: ____

Initial Value: $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, IVP solution: ________________________

For this IVP solution:

$x_1(t) =$ ____

$x_2(t) =$ ____

Solve for x_1 in terms of x_2:

Answer the following questions for $A = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix}$:

The smaller eigenvalue of A is $r_1 =$ ____. An eigenvector corresponding to r_1 is $\mathbf{v} =$

The larger eigenvalue of A is $r_2 =$ ____. An eigenvector corresponding to r_2 is $\mathbf{w} =$

The general solution to $\mathbf{x}' = A\mathbf{x}$ is

For large **positive** values of t which is larger: $e^{r_1 t}$ or $e^{r_2 t}$?

For the following problems, consider the case when $c_1 \neq 0$ and $c_2 \neq 0$ where the general solution is $\mathbf{x} = c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t} + c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$,

For large **positive** values of t, which term dominates: $c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t}$ or $c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$?

Thus for large **positive** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

* moves away from the origin.
* moves toward the origin.
* approaches the line $y = mx$ with slope $m =$ ______________
* approaches a line $y = mx + b$ for $b \neq 0$ with slope $m =$ ______________. Note this case corresponds to where both $||c_1 \mathbf{v}||e^{r_1 t}$ and $||c_2 \mathbf{w}||e^{r_2 t}$ are large, but one is significantly larger than the other.

For large **negative** values of t which is larger: $e^{r_1 t}$ or $e^{r_2 t}$?

For large **negative** values of t, which term dominates: $c_1 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{r_1 t}$ or $c_2 \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} e^{r_2 t}$?

Thus for large **negative** values of t, such trajectories (where $c_1 c_2 \neq 0$) when projected into the x_1, x_2 plane exhibit the following behavior (select all that apply):

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Give that the solution to $\mathbf{x}' = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix} \mathbf{x}$ is $\mathbf{x} = c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ in the

t, x_1-plane t, x_2-plane x_1, x_2-plane

Graph the solution to the IVP $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ in the

t, x_1-plane t, x_2-plane x_1, x_2-plane

The equilibrium solution for this system of equations is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$.

$\frac{dx_2}{dx_1} =$ ________________

Plot several direction vectors where the slope is 0 and where slope is vertical.

Graph several trajectories.

Find the **equilibrium solution(s)** for $\mathbf{x}' = A\mathbf{x}$

Recall a solution is an equilibrium solution iff $\mathbf{x}(t) = \mathbf{C}$ iff $\mathbf{x}'(t) = 0$

Setting $\mathbf{x}' = 0$, implies $\mathbf{0} = A\mathbf{x}$.

Thus $\mathbf{x} = \mathbf{C}$ is an equilibrium solution iff it is a solution to $\mathbf{0} = A\mathbf{x}$.

<u>Case 1</u> (not emphasized/covered): $det(A) = 0$.

In this case, $A\mathbf{x} = \mathbf{0}$ has an infinite number of solutions. Note this case corresponds to the case when 0 is an eigenvalue of A since there are nonzero solutions to $A\mathbf{v} = 0\mathbf{v}$

<u>Case 2</u>: $det(A) \neq 0$.

Then $A\mathbf{x} = \mathbf{0}$ has a unique solution, $\mathbf{x} =$

Thus if $det(A) \neq 0$, $\mathbf{x} =$ is the only equilibrium solution of $\mathbf{x}' = A\mathbf{x}$

Slope fields:

* For complex eigenvalue case, one slope is needed.

* For real eigenvalue case, 0 and ∞ slopes can be helpful and can catch graphing errors, but your graph does **not** need to be that accurate.

For $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 21 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$

$\frac{dx_1}{dt} =$

$\frac{dx_2}{dt} =$

$\frac{dx_2}{dx_1} =$

Slope 0:

Slope ∞:

For $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ -9 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$

$$\frac{dx_1}{dt} =$$

$$\frac{dx_2}{dt} =$$

$$\frac{dx_2}{dx_1} =$$

Slope 0:

Slope ∞:

For $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 9 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$

$$\frac{dx_1}{dt} =$$

$$\frac{dx_2}{dt} =$$

$$\frac{dx_2}{dx_1} =$$

Slope 0:

Slope ∞: