Formula 13: $\mathcal{L}(u_c(t)f(t-c)) = e^{-cs}\mathcal{L}(f(t))$.

or equivalently

$$\mathcal{L}(u_c(t)f(t)) = e^{-cs}\mathcal{L}(f(t+c)).$$

---

a.) $\mathcal{L}(u_3(t)(t^2-2t+1)) = \underline{e^{-3s}(\frac{2}{s^3}+\frac{4}{s^2}+\frac{4}{s})}$

$$\mathcal{L}(u_3(t)(t^2-2t+1)) = e^{-3s}\mathcal{L}((t+3)^2-2(t+3)+1))$$

$$= e^{-3s}\mathcal{L}(t^2+6t+9-2t-6+1))$$

$$= e^{-3s}\mathcal{L}(t^2+4t+4) = e^{-3s}(\frac{2}{s^3}+\frac{4}{s^2}+\frac{4}{s})$$

---

b.) $\mathcal{L}(u_4(t)(e^{-8t})) = \underline{e^{-4s-32}\left(\frac{1}{s+8}\right)}$

$$\mathcal{L}(u_4(t)(e^{-8t})) = e^{-4s}\mathcal{L}(e^{-8(t+4)}) = e^{-4s}\mathcal{L}(e^{-8t}e^{-32}))$$

$$= e^{-4s}e^{-32}\mathcal{L}(e^{-8t}) = e^{-4s-32}\left(\frac{1}{s+8}\right)$$

Find the LaPlace transform of

d.) $g(t) = \begin{cases} 0 & t < 3 \\ e^{t-3} & t \geq 3 \end{cases}$

Note $g(t) = u_3(t)e^{t-3}$

$$\mathcal{L}(u_3(t)e^{t-3}) = e^{-3s}\mathcal{L}(e^t) = \frac{e^{-3s}}{s-1}$$

c.) $\mathcal{L}(u_2(t)(t^2e^{3t})) = \underline{e^{-2s+6}(\frac{2}{(s-3)^3} + \frac{4}{(s-3)^2} + \frac{4}{(s-3)})}$

$$\mathcal{L}(u_2(t^2e^{3t})) = e^{-2s}\mathcal{L}([(t+2)^2]e^{3(t+2)})$$

$$= e^{-2s}\mathcal{L}([t^2+4t+4]e^{3t+6})$$

$$= e^{-2s}e^6\mathcal{L}([t^2+4t+4]e^{3t})$$

$$= e^{-2s+6}\mathcal{L}(t^2e^{3t} + 4te^{3t} + 4e^{3t})$$

$$= e^{-2s+6}(\mathcal{L}(t^2e^{3t}) + 4\mathcal{L}(te^{3t}) + 4\mathcal{L}(e^{3t}))$$

$$= e^{-2s+6}(\tfrac{2}{(s-3)^3} + \tfrac{4}{(s-3)^2} + \tfrac{4}{(s-3)}) \text{ since}$$

Formula 14: $\mathcal{L}(e^{cs}f(t)) = F(s-c)$

Thus $\mathcal{L}(t^2e^{3t}) = F(s-3) = \frac{2}{(s-3)^3}$

since $F(s) = \mathcal{L}(f(t)) = \mathcal{L}(t^2) = \frac{2}{s^3}$

and $F(s-3) = \frac{2}{(s-3)^3}$

Similarly, $\mathcal{L}(te^{3t}) = \frac{1}{(s-3)^2}$

---

e.) $f(t) = \begin{cases} 0 & t < 3 \\ 5 & 3 \le t < 4 \\ t-5 & t \ge 4 \end{cases}$

$$f(t) = 0 + u_3(t)[5-0] + u_4(t)[t-5-5]$$

$$\mathcal{L}(f(t)) = \mathcal{L}(5u_3(t) + u_4(t)[t-10])$$

$$= 5\mathcal{L}(u_3(t)) + \mathcal{L}(u_4(t)[t-10])$$

$$= 5e^{-3s} + e^{-4s}\mathcal{L}(t+4-10)$$

$$= 5e^{-3s} + e^{-4s}\mathcal{L}(t-6)$$

$$= 5e^{-3s} + e^{-4s}[\mathcal{L}(t) - 6\mathcal{L}(1)]$$

$$= 5e^{-3s} + e^{-4s}[\tfrac{1}{s^2} - \tfrac{6}{s}] = 5e^{-3s} + \tfrac{e^{-4s}(1-6s)}{s^2}$$

Formula 13: $\mathcal{L}(u_c(t)f(t-c)) = e^{-cs}\mathcal{L}(f(t))$.

Let $F(s) = \mathcal{L}(f(t))$.

Then $\mathcal{L}^{-1}(F(s)) = \mathcal{L}^{-1}(\mathcal{L}(f(t))) = f(t)$.

Thus $\mathcal{L}^{-1}(e^{-cs}F(s))$

$$= \mathcal{L}^{-1}(e^{-cs}\mathcal{L}(f(t))) = u_c(t)f(t-c)$$

where $f(t) = \mathcal{L}^{-1}(F(s))$

---

a.) $\mathcal{L}^{-1}(e^{-8s}\frac{1}{s-3}) = \underline{u_8(t)e^{3(t-8)}}$

$\mathcal{L}^{-1}(e^{-8s}\frac{1}{s-3}) = u_8(t)f(t-8)$ where

$\mathcal{L}(f(t)) = \frac{1}{s-3}$. Hence $f(t) = \mathcal{L}^{-1}(\frac{1}{s-3}) = e^{3t}$

b.) $\mathcal{L}^{-1}(e^{-4s}\frac{1}{s^2-3}) = \underline{u_4(t)\frac{1}{\sqrt{3}}\ sinh(\sqrt{3}(t-4))}$

$\mathcal{L}^{-1}(e^{-4s}\frac{1}{s^2-3}) = u_4(t)f(t-4)$ where

$\mathcal{L}(f(t)) = \frac{1}{s^2-3}$. Hence $f(t) = \frac{1}{\sqrt{3}}\mathcal{L}^{-1}(\frac{\sqrt{3}}{s^2-3}) = \frac{1}{\sqrt{3}}sinh(\sqrt{3}t)$

---

c.) $\mathcal{L}^{-1}(e^{-s}\frac{5}{(s-3)^4}) = \underline{u_1(t)(\frac{5}{6})(t-1)^3e^{3(t-1)}}$

$\mathcal{L}^{-1}(e^{-s}\frac{5}{(s-3)^4}) = u_1(t)f(t-1)$ where

$\mathcal{L}(f(t)) = \frac{5}{(s-3)^4}$. Hence $f(t) = \frac{5}{6}\mathcal{L}^{-1}(\frac{3!}{(s-3)^4}) = \frac{5}{6}t^3e^{3t}$

---

d.) $\mathcal{L}^{-1}(\frac{e^{-s}}{4s}) = \underline{\frac{1}{4}u_1(t)}$

In this case you can use the easier formula 12, or alternatively, you can use formula 13 (but formula 12 is easier to use and applies to this case):

$\mathcal{L}^{-1}(\frac{e^{-s}}{4s}) = \frac{1}{4}\mathcal{L}^{-1}(\frac{e^{-s}}{s}) = \frac{1}{4}u_1(t)f(t+1)$ where

$\mathcal{L}(f(t)) = \frac{1}{s}$. Hence $f(t) = 1$. Thus $f(t-1) = 1$

---

e.) $\mathcal{L}^{-1}(e^{-s}) = \underline{\delta(t-1)}$

f.) $\mathcal{L}^{-1}(e^{-s}\frac{1}{(s-3)^2+4}) = \underline{\frac{1}{2}u_1(t)e^{3(t-1)}sin(2(t-1))}$

$\mathcal{L}^{-1}(e^{-s}\frac{1}{(s-3)^2+4}) = u_1(t)f(t-1)$ where

$\mathcal{L}(f(t)) = \frac{1}{(s-3)^2+4})$.

Hence $f(t) = \frac{1}{2}\mathcal{L}^{-1}(\frac{2}{(s-3)^2+4}) = \frac{1}{2}e^{3t}sin(2t)$

---

g.) $\mathcal{L}^{-1}(e^{-s}\frac{2s-5}{s^2+6s+13})$

$$= \underline{u_1(t)e^{-3t+1}[2cos(2t-2) - \frac{11}{2}sin(2t-2)]}$$

$$\frac{2s-5}{s^2+6s+13} = \frac{2s-5}{s^2+6s+9-9+13} = \frac{2s-5}{(s+3)^2+4} = \frac{2(s+3)-6-5}{(s+3)^2+4}$$

$$\mathcal{L}^{-1}(\frac{2s-5}{s^2+6s+13}) = \mathcal{L}^{-1}(\frac{2(s+3)-11}{(s+3)^2+4})$$

$$= 2\mathcal{L}^{-1}(\frac{s+3}{(s+3)^2+4}) - 11\mathcal{L}^{-1}(\frac{1}{(s+3)^2+4})$$

$$= 2\mathcal{L}^{-1}(\frac{s+3}{(s+3)^2+4}) - \frac{11}{2}\mathcal{L}^{-1}(\frac{2}{(s+3)^2+4})$$

$$= 2e^{-3t}cos(2t) - \frac{11}{2}e^{-3t}sin(2t)$$

$$\mathcal{L}^{-1}(e^{-s}\frac{2s-5}{s^2+6s+13}) = u_1(t)f(t-1)$$

$$= u_1(t)[2e^{-3(t-1)}cos(2(t-1)) - \frac{11}{2}e^{-3(t-1)}sin(2(t-1))]$$

$$= u_1(t)e^{-3t+1}[2cos(2t-2) - \frac{11}{2}sin(2t-2)]$$