Section 6.3

Example: $f(t) = \begin{cases} f_1, & \text{if } t < 4; \\ f_2, & \text{if } 4 \leq t < 5; \\ f_3, & \text{if } 5 \leq t < 10; \\ f_4, & \text{if } t \geq 10; \end{cases}$

Hence $f(t) = f_1(t) + u_4(t)[f_2(t) - f_1(t)] + u_5(t)[f_3(t) - f_2(t)] + u_{10}(t)[f_4(t) - f_3(t)]$

Formula 13: $\mathcal{L}(u_c(t)f(t-c)) = e^{-cs}\mathcal{L}(f(t))$.

or equivalently

$$\mathcal{L}(u_c(t)f(t-c+c)) = e^{-cs}\mathcal{L}(f(t+c)).$$

or equivalently

$$\mathcal{L}(u_c(t)f(t)) = e^{-cs}\mathcal{L}(f(t+c)).$$

In other words, replacing $t-c$ with t is equivalent to replacing t with $t+c$

Formula 13: $\mathcal{L}(u_c(t)f(t-c)) = e^{-cs}\mathcal{L}(f(t))$.

Let $F(s) = \mathcal{L}(f(t))$. Then $\mathcal{L}^{-1}(F(s)) = \mathcal{L}^{-1}(\mathcal{L}(f(t))) = f(t)$.

Thus $\mathcal{L}^{-1}(e^{-cs}F(s)) = \mathcal{L}^{-1}(e^{-cs}\mathcal{L}(f(t))) = u_c(t)f(t-c)$ where $f(t) = \mathcal{L}^{-1}(F(s))$ $\blacksquare$