6.3: Step functions.

$$u_c(t) = \begin{cases} 0 & t < c \\ 1 & t \geq c \end{cases}$$

1.) Graph $u_c(t)$:

2.) Given f, graph $u_c(t)f(t-c)$:

Example: $f(t) = \begin{cases} f_1, & \text{if } t < 4; \\ f_2, & \text{if } 4 \le t < 5; \\ f_3, & \text{if } 5 \le t < 10; \\ f_4, & \text{if } t \ge 10; \end{cases}$

Hence

$$f(t) = f_1(t) + u_4(t)[f_2(t) - f_1(t)] + u_5(t)[f_3(t) - f_2(t)] \\ + u_{10}(t)[f_4(t) - f_3(t)]$$

3.) Calculate $\mathcal{L}(u_c(t)f(t-c))$ in terms of $\mathcal{L}(f(t))$:

Example: Find the LaPlace transform of

4.) $$g(t) = \begin{cases} 0 & t < 3 \\ e^{t-3} & t \geq 3 \end{cases}$$

5.) $$f(t) = \begin{cases} 0 & t < 3 \\ 5 & 3 \leq t < 4 \\ t - 5 & t \geq 4 \end{cases}$$

6.) Ex: Find the inverse Laplace transform of $\frac{e^{-8s}}{s^3}$

7.) Calculate $\mathcal{L}(e^{ct}f(t))$ in terms of $F(s) = \mathcal{L}(f(t))$

8.) Example: Use formula 6 (p. 317) to find the inverse LaPlace transform of $\frac{s-c}{(s-c)^2+a^2}$.

Formula 13: $\mathcal{L}(u_c(t)f(t-c)) = e^{-cs}\mathcal{L}(f(t))$.

or equivalently

$$\mathcal{L}(u_c(t)f(t-c+c)) = e^{-cs}\mathcal{L}(f(t+c)).$$

or equivalently

$$\mathcal{L}(u_c(t)f(t)) = e^{-cs}\mathcal{L}(f(t+c)).$$

In other words, replacing $t-c$ with t is equivalent to replacing t with $t+c$

Formula 13: $\mathcal{L}(u_c(t)f(t-c)) = e^{-cs}\mathcal{L}(f(t))$.

Let $F(s) = \mathcal{L}(f(t))$.

Then $\mathcal{L}^{-1}(F(s)) = \mathcal{L}^{-1}(\mathcal{L}(f(t))) = f(t)$.

Thus $\mathcal{L}^{-1}(e^{-cs}F(s))$

$$= \mathcal{L}^{-1}(e^{-cs}\mathcal{L}(f(t))) = u_c(t)f(t-c)$$

where $f(t) = \mathcal{L}^{-1}(F(s))$