$$\mathcal{L}(f'(t)) = \int_0^{\infty} e^{-st} f'(t)dt = \lim_{A\to\infty} \int_0^{A} e^{st} f'(t)dt$$

Integration by parts: Let $u = e^{-st}$ $\quad dv = f'(t)dt$

Then $du = -se^{st}dt \quad v = f(t)$

$$\int_0^A e^{-st} f'(t)dt = e^{-st} f(t)|_0^A - \int_0^A [-se^{-st} f(t)]dt$$

$$= e^{-sA} f(A) - f(0) + s \int_0^A e^{-st} f(t)dt$$

Thus $\mathcal{L}(f'(t)) = \lim_{A\to\infty} \int_0^A e^{st} f'(t)dt$

$$= \lim_{A\to\infty} e^{-sA} f(A) - f(0) + s \int_0^A e^{-st} f(t)dt$$

$$= 0 - f(0) + s \int_0^{\infty} e^{-st} f(t)dt$$

$$= 0 - f(0) + s\mathcal{L}(f(t))$$

$$= s\mathcal{L}(f(t)) - f(0)$$

Thus $\mathcal{L}(f'(t)) = s\mathcal{L}(f(t)) - f(0)$

and $\mathcal{L}(f''(t)) = s\mathcal{L}(f'(t)) - f'(0) = s[s\mathcal{L}(f(t)) - f(0)] - f'(0)$

$$= s^2\mathcal{L}(f(t)) - sf(0) - f'(0)$$

and $\mathcal{L}(f'''(t)) = s\mathcal{L}(f''(t)) - f''(0)$

$$= s[s^2\mathcal{L}(f(t)) - sf(0) - f'(0)] - f''(0)$$

$$= s^3\mathcal{L}(f(t)) - s^2 f(0) - sf'(0) - f''(0)$$

etc. And thus

$\mathcal{L}(f^{(n)}(t)) = s^n\mathcal{L}(f(t)) - s^{n-1}f(0) - s^{n-2}f'(0) - ... - f^{(n-1)}(0)$