Ex: Find the Wronskian of a fundamental set of solutions of the DE

$y'' + 5y' = 0$

Method 1: Find homogeneous solution

$r^2 + 5r = 0$ implies $r = 0, -5$

homog sol'n $y = c_1 e^{0t} + c_2 e^{-5t} = c_1(1) + c_2 e^{-5t} = c_1 + c_2 e^{-5t}$

A fundamental set of solutions: $\{1, e^{-5t}\}$

Wronskian $= W(1, e^{-5t})(t) = det \begin{pmatrix} 1 & e^{-5t} \\ 0 & -5e^{-5t} \end{pmatrix} = -5e^{-5t}$

Method 2: Abel's theorem: Wronskian $= ce^{-\int p_1(t)dt}$

$y'' + 5y' = 0$ implies $p_1(t) = 5$.

Thus Wronskian $= W(1, e^{-5t})(t) = ce^{-\int 5dt} = ce^{-5t}$

The above is what the book is looking for, but we can find c.

$W(\phi 1, \phi 2)(0) = det \begin{pmatrix} 1 & 1 \\ 0 & -5 \end{pmatrix} = -5$

$W(\phi 1, \phi 2)(0) = ce^0 = c$

Thus $c = -5$

Thus $W(1, e^{-5t})(t) = -5e^{-5t}$