

Guess a possible non-homog soln for the following DEs:

1a.) $y'' - 4y' - 9y = 4\cos(3t)$

Guess: $y = A\cos(3t) + B\sin(3t)$

1b.) $y'' - 9y = 4\cos(3t)$ *no y' term so don't need $B\sin(3t)$ term*

Guess: $y = A\cos(3t)$ or $y = A\cos(3t) + B\sin(3t)$ *$B=0$*

since no y' term, do not need $B\sin(3t)$ term as no $\sin(3t)$ will appear on LHS when plug in guess. $\Rightarrow B=0$

1c.) $y'' - 9y' = 4\cos(3t)$

Guess: $y = A\cos(3t) + B\sin(3t)$

1d.) $y'' + 9y = 4\cos(9t)$

Guess: $y = A\cos(9t)$ since no y' term

If guess is homogeneous multiply non simplified guess by t^k due to product rule

1e.) $y'' + 9y = 4\cos(3t)$

Guess: $y = t \cdot (A\cos(3t) + B\sin(3t))$

Wrong guess $y = A\cos(3t)$ since it is a homog soln:

Solve homogeneous first $r^2 + 9 = 0 \Rightarrow r = \pm 3i$ *If plus in, get 0*

homog soln $y = c_1 \cos(3t) + c_2 \sin(3t)$

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1a.) $y'' - 4y' - 9y = 4\cos(3t)$

Guess: $y = A\cos(3t) + B\sin(3t)$

1b.) $y'' - 9y = 4\cos(3t)$ $\Rightarrow B=0$

Guess: $y = A\cos(3t) + B\sin(3t)$ or $y = A\cos(3t)$

since no y' term, do not need $B\sin(3t)$ term as no $\sin(3t)$ will appear on LHS when plug in guess.

1c.) $y'' - 9y' = 4\cos(3t)$

Guess: $y = A\cos(3t) + B\sin(3t)$

1d.) $y'' + 9y = 4\cos(9t)$

Guess: $y = A\cos(9t)$ since no y' term don't need $B\sin(9t)$

1e.) $y'' + 9y = 4\cos(3t)$ \rightarrow product rule $\Rightarrow \sin(3t)$ term will appear on LHS when plug in $tA\cos(3t)$

Guess: $y = t \cdot (A\cos(3t) + B\sin(3t))$

Wrong guess: $y = A\cos(3t)$ since it is a homog soln
plugin \Rightarrow get 0

solve homog: $r^2 + 9 = 0 \Rightarrow r = \pm 3i$

homog soln $y = c_1 \cos(3t) + c_2 \sin(3t)$

$$2a.) y'' + 9y = [4\cos(9t)] + (t)$$

Guess: $y = A_1 \cos 9t + A_2 t$
 don't need $+ B_1 \sin 9t$ since no y' term
 don't need $+ B_2 t$ term since no y^2 term

$$2b.) y'' + 9y = [4\cos(3t)] + (t)$$

Guess: $y = t \cdot (A \cos(3t) + B \sin(3t)) + A_2 t$
 homog \uparrow need due to product rule

$$2c.) y'' + 9y = 4t \cos(9t)$$

Guess: $y = A t \cos(9t) + B t \sin(9t) + C \cos(9t) + D \sin(9t)$

Alternatively, plug in wrong guess and see what is missing.

$y = t \cdot A \cos(9t) \leftarrow$ wrong guess
 $\star \star \star$ product rule \Rightarrow use non-simplified guesses $\star \star \star$

$$y = (A_1 t + B_1) \cdot (A_2 \cos(9t) + B_2 \sin(9t))$$

$$y = A A_2 t \cos(9t) + A_1 B_2 t \sin(9t) + B_1 A_2 \cos(9t) + B_1 B_2 \sin(9t)$$

$y = A t \cos 9t + B t \sin(9t) + C \cos(9t) + D \sin(9t)$
 plug in entire guess to solve for all 4 unknowns

$$2d.) y'' + 9y = 4t \cos(3t)$$

Guess: $y = t \cdot (A t \cos(3t) + B t \sin(3t) + C \cos(3t) + D \sin(3t))$
 \star since $y = \cos(3t)$ is homog sol
 we multiply non-simplified guess by t