Section 2.7 Euler method: Using tangent lines to approximate a function.

$y_{i+1} = y_i + \Delta y = y_i + \frac{\Delta y}{\Delta t}\Delta t \cong y_i + \frac{dy}{dt}\Delta t$

Alternatively use equation of tangent line:

$$\text{slope} = \frac{y_{i+1}-y_i}{t_{i+1}-t_i} = f'(y_i, t_i).$$

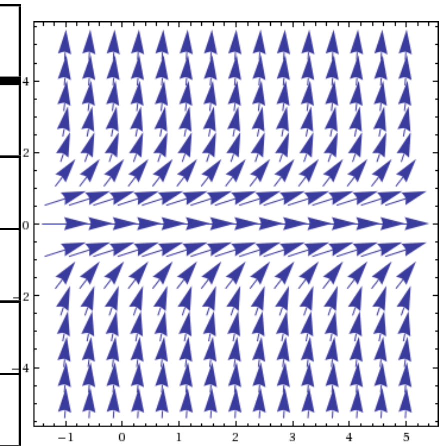
$y_{i+1} = f'(y_i, t_i)(t_{i+1} - t_i) + y_i = T(t_{i+1})$ where $y = T(t)$ is the equation of the tangent line at (y_i, t_i).

Example: $\frac{dy}{dt} = y^2$, $y(2) = 1$ implies $y = \frac{1}{3-t}$.

t	$y = 1/(3-t)$	approximation
2.000000	1.000000	1.000000
3.000000	999.000000	2.000000
4.000000	-1.000000	6.000000
5.000000	-0.500000	42.000000
6.000000	-0.333333	1806.000000

t	$y = 1/(3-t)$	approximation
2.000000	1.000000	1.000000
2.100000	1.111111	1.100000
2.200000	1.250000	1.221000
2.300000	1.428571	1.370084
2.400000	1.666667	1.557797
2.500000	2.000000	1.800470
2.600000	2.500000	2.124640
2.700000	3.333333	2.576049
2.800000	5.000000	3.239652
2.900000	10.000004	4.289186

t	$y = 1/(3-t)$	approximation
2.00	1.000000	1.000000
2.01	1.010101	1.010000
2.02	1.020408	1.020201
2.03	1.030928	1.030609
2.04	1.041667	1.041231
2.05	1.052632	1.052072
2.06	1.063830	1.063141
2.07	1.075269	1.074443
2.08	1.086957	1.085988
2.09	1.098901	1.097782
2.10	1.111111	1.109833
2.11	1.123595	1.122150
2.12	1.136364	1.134742
2.13	1.149425	1.147619
2.87	7.692308	6.721314
2.88	8.333333	7.173075
2.89	9.090908	7.687605
2.90	9.999998	8.278598
2.91	11.111107	8.963949
2.92	12.499993	9.767473
2.93	14.285716	10.721509
2.94	16.666666	11.871017
2.95	19.999996	13.280227
2.96	24.999987	15.043871
2.97	33.333298	17.307051
2.98	49.999897	20.302391
2.99	99.999496	24.424261



$y' = y^2$