Section 2.4 $y' = y^{1/3}$

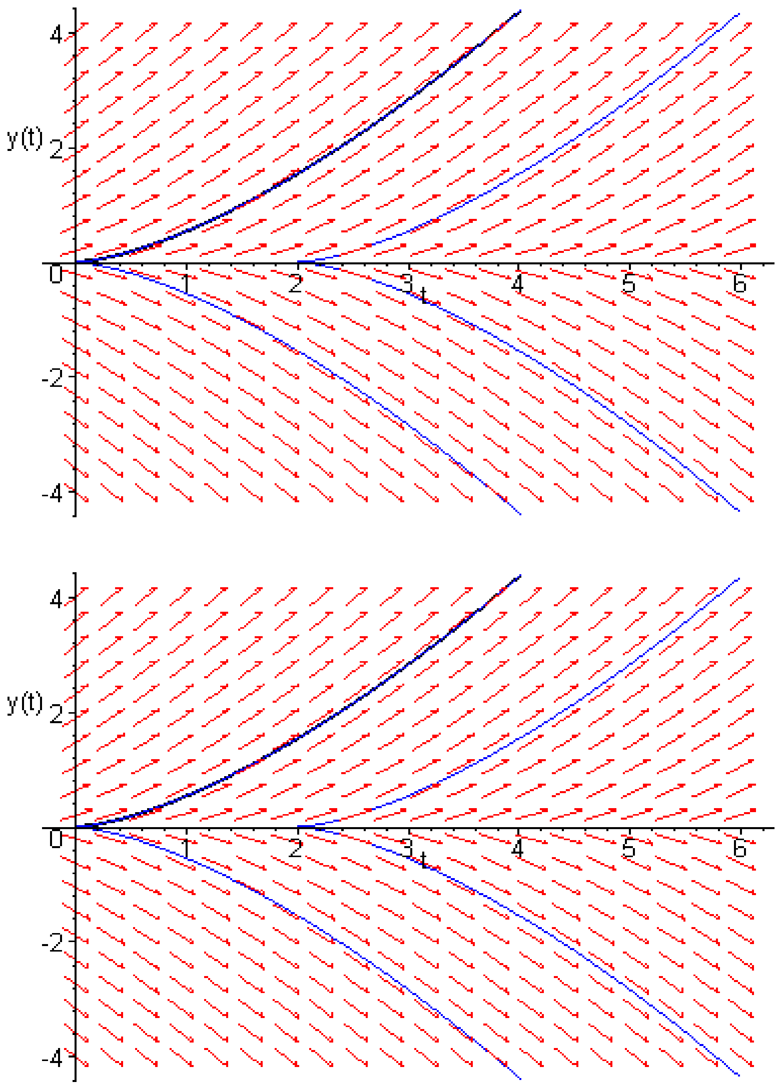


Figure 2.4.1 from ***Elementary Differential Equations and Boundary Value Problems*, Eighth Edition** by William E. Boyce and Richard C. DiPrima

Note IVP, $y' = y^{\frac{1}{3}}$, $y(x_0) = 0$ has an infinite number of solutions,

while IVP, $y' = y^{\frac{1}{3}}$, $y(x_0) = y_0$ where $y_0 \neq 0$ has a unique solution.

Initial Value Problem: $y(t_0) = y_0$
Use initial value to solve for C.

Examples: No solution:

Ex 1: $y' = y' + 1$

Ex 2: $(y')^2 = -1$

Ex 3 (IVP): $\frac{dy}{dx} = y(1 + \frac{1}{x})$, $y(0) = 1$

$\int \frac{dy}{y} = \int (1 + \frac{1}{x})dx \quad$ implies $\quad ln|y| = x + ln|x| + C$

$|y| = e^{x+ln|x|+C} = e^x e^{ln|x|} e^C = C|x|e^x = Cxe^x$

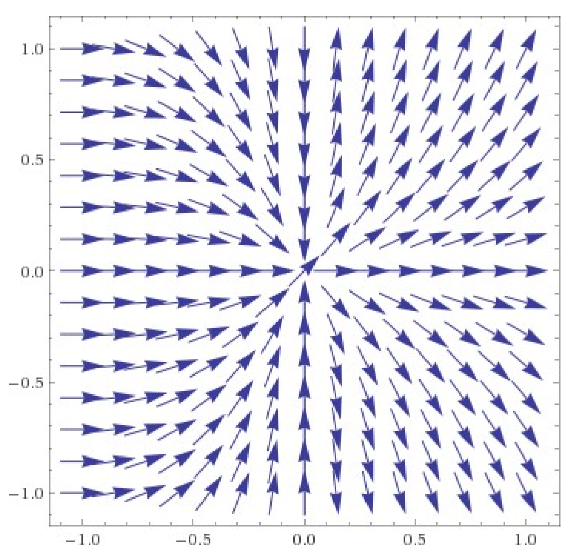
$y = \pm Cxe^x$ implies $y = Cxe^x$

$y(0) = 1: \quad 1 = C(0)e^0 = 0$ implies

IVP $\frac{dy}{dx} = y(1 + \frac{1}{x})$, $y(0) = 1$ has no solution.

http://www.wolframalpha.com

slope field: $\{1, y(1+1/x)\}/sqrt(1+y\wedge 2(1+1/x)\wedge 2)$



Special cases:

Suppose f is cont. on (a, b) and the point $t_0 \in (a, b)$, Solve IVP: $\frac{dy}{dt} = f(t)$, $y(t_0) = y_0$

$$dy = f(t)dt$$

$$\int dy = \int f(t)dt$$

$y = F(t) + C$ where F is any anti-derivative of F.

Initial Value Problem (IVP): $y(t_0) = y_0$

$$y_0 = F(t_0) + C \text{ implies } C = y_0 - F(t_0)$$

Hence unique solution (if domain connected) to IVP:

$$y = F(t) + y_0 - F(t_0)$$

First order linear differential equation:

Thm 2.4.1: If p and g are continuous on (a, b) and the point $t_0 \in (a, b)$, then there exists a unique function $y = \phi(t)$ defined on (a, b) that satisfies the following initial value problem:

$$y' + p(t)y = g(t), \quad y(t_0) = y_0.$$

More general case (but still need hypothesis)

Thm 2.4.2: Suppose the functions $z = f(t, y)$ and $z = \frac{\partial f}{\partial y}(t, y)$ are continuous on $(a, b) \times (c, d)$ and the point $(t_0, y_0) \in (a, b) \times (c, d)$,

then there exists an interval $(t_0 - h, t_0 + h) \subset (a, b)$ such that there exists a unique function $y = \phi(t)$ defined on $(t_0 - h, t_0 + h)$ that satisfies the following initial value problem:

$$y' = f(t, y), \quad y(t_0) = y_0.$$

If possible **without solving**, determine where the solution exists for the following initial value problems: