MATH:7450 (22M:305) Topics in Topology: Scientific and Engineering Applications of Algebraic Topology

Sept 16, 2013: Persistent homology III

Fall 2013 course offered through the
University of Iowa Division of Continuing Education

Isabel K. Darcy, Department of Mathematics
Applied Mathematical and Computational Sciences,
University of Iowa

http://www.math.uiowa.edu/~idarcy/AppliedTopology.html

http://homepage.math.uiowa.edu/~idarcy/AT/schedule.html

Sept 11 video, pptx, pdf	Persistent homology Topology and data, G Carlsson (2009) H. Edelsbrunner, D. Letscher, and A. Zomorodian, Topological persistence and simpliction, Discrete and Computational Geometry 28, 2002, 511-533. Ch 7 in Afra J. Zomorodian, Topology for Computing (Cambridge Monographs on Applied and Computational Mathematics), Cambridge University Press (September 28, 2009) google books preview
Sept 13 video, pptx, pdf	Persistance homology (cont.)
	Additional readings Klein bottle Topological analysis of population activity in visual cortex, Gurjeet Singh, Facundo Memoli, Tigran Ishkhanov, Guillermo Sapiro, Gunnar Carlsson, Dario L. Ringach, J Vis. 2008 Jun 30;8(8):11.1-18
Week 4	
Sept 16 pptx, pdf	Javaplex Javaplex tutorial

Discrete Comput Geom 33:249–274 (2005)
DOI: 10.1007/s00454-004-1146-y

Discrete & Computational
Geometry

http://link.springer.com/article/10.1007%2Fs00454-004-1146-y

Computing Persistent Homology*

Afra Zomorodian[1] and Gunnar Carlsson[2]

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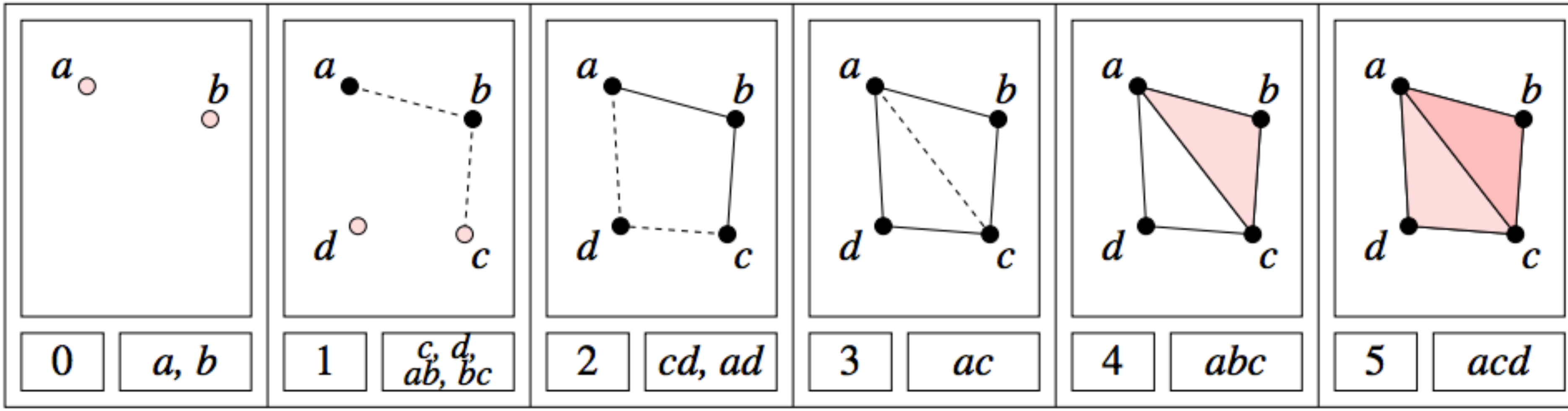
Computing Persistent Homology by Afra Zomorodian, Gunnar Carlsson



$$C_1 \xrightarrow{\partial_1} C_0$$

$$M_1 = \left[\begin{array}{c|ccccc} & ab & bc & cd & ad & ac \\ \hline d & 0 & 0 & t & t & 0 \\ c & 0 & 1 & t & 0 & t^2 \\ b & t & t & 0 & 0 & 0 \\ a & t & 0 & 0 & t^2 & t^3 \end{array}\right]$$

http://link.springer.com/article/10.1007%2Fs00454-004-1146-y

$$
\begin{array}{c|ccccc}
 & cd & bc & ab & ad + cd + t \cdot bc + t \cdot ab & ac + t^2 \cdot bc + t^2 \cdot ab \\
\hline
d & t & 0 & 0 & 0 & 0 \\
c & t & 1 & 0 & 0 & 0 \\
b & 0 & t & t & 0 & 0 \\
a & 0 & 0 & t & 0 & 0
\end{array}
$$

B_0 = image of ∂_1 = column space of M_1

$= \langle tc + td,\ tb + c,\ ta + tb \rangle$

Z_1 = kernel of ∂_1 = null space of M_1

$= \langle ad + cd + t(bc) + t(ab),\ ac + t^2bc + t^2ab \rangle$

$$C_1 \xrightarrow{\partial_1} C_0 \xrightarrow{\partial_0} 0$$

$$H_0 = Z_0/B_0 = (\text{kernel of } \partial_0)/\ (\text{image of } \partial_1)$$

$$= \frac{\text{null space of } M_0}{\text{column space of } M_1}$$

$$= \langle a, b, c, d \ :\ tc + td,\ tb + c,\ ta + tb \rangle$$

$$
\begin{array}{c} \\ d \\ c \\ b \\ a \end{array}
\begin{array}{c}
\begin{array}{ccccc} cd & bc & ab & ad + cd + t \cdot bc + t \cdot ab & ac + t^2 \cdot bc + t^2 \cdot ab \end{array} \\
\begin{pmatrix} t & 0 & 0 & 0 & 0 \\ t & 1 & 0 & 0 & 0 \\ 0 & t & t & 0 & 0 \\ 0 & 0 & t & 0 & 0 \end{pmatrix}
\end{array}
$$

B_0 = image of ∂_1 = column space of M_1

= < tc + td, tb + c, ta + tb>

Z_1 = kernel of ∂_1 = null space of M_1

= < ad + cd + t(bc) + t(ab), ac + t^2bc + t^2ab >

$$C_2 \xrightarrow{\partial_1} C_1 \xrightarrow{\partial_1} C_0$$

$H_1 = Z_1/B_1 =$ (kernel of ∂_1)/ (image of ∂_2)

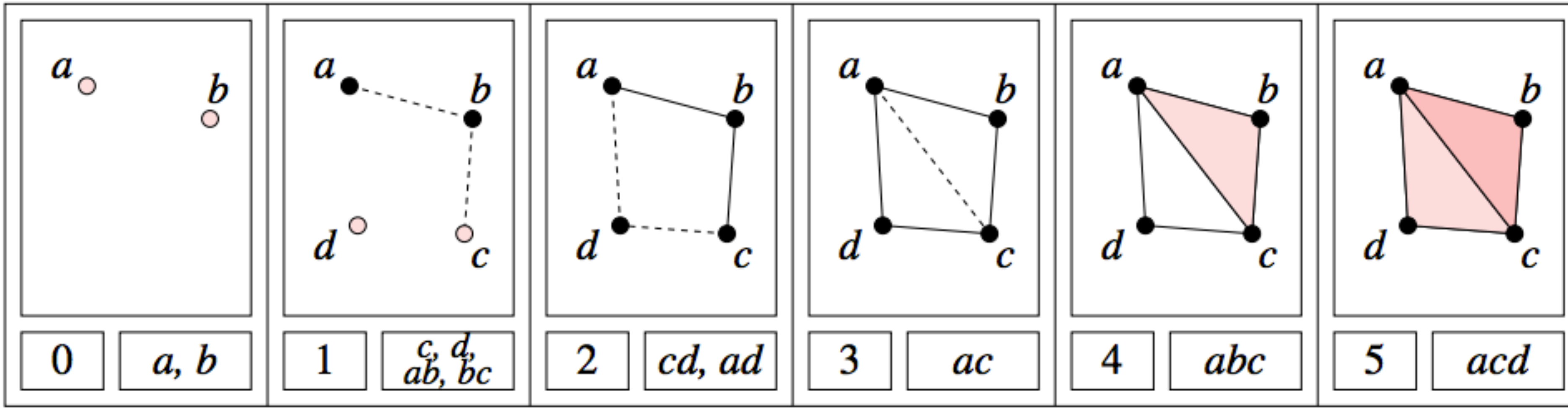
$$= \frac{\text{null space of } M_1}{\text{column space of } M_2}$$

$$= < ad + cd + t(bc) + t(ab),\ ac + t^2bc + t^2ab \ : \ ??????? >$$

Let $z_1 = ad + cd + t(bc) + t(ab)$, $z_2 = ac + t^2bc + t^2ab$

$$H_1 = Z_1/B_1 = <z_1, z_2 \ : \ ?? >$$

Computing Persistent Homology by Afra Zomorodian, Gunnar Carlsson



$$C_2 \xrightarrow{\partial_2} C_1$$

$$M_2 = \begin{array}{c} \\ cd \\ bc \\ ab \\ ad \\ ac \end{array} \begin{array}{c} \begin{array}{cc} abc & acd \end{array} \\ \begin{pmatrix} 0 & t^3 \\ t^3 & 0 \\ t^3 & 0 \\ 0 & t^3 \\ t & t^2 \end{pmatrix} \end{array}$$

http://link.springer.com/article/10.1007%2Fs00454-004-1146-y

Long method for determining column space of M_2

$$\begin{array}{c} \\ cd \\ bc \\ ab \\ ad \\ ac \end{array} \begin{array}{c} \begin{array}{cc} abc & acd \end{array} \\ \begin{pmatrix} 0 & t^3 \\ t^3 & 0 \\ t^3 & 0 \\ 0 & t^3 \\ t & t^2 \end{pmatrix} \end{array}$$

$$\begin{array}{c} \\ cd \\ bc \\ ab \\ ad + cd \\ ac \end{array} \begin{array}{c} \begin{array}{cc} abc & acd \end{array} \\ \begin{pmatrix} 0 & 0 \\ t^3 & 0 \\ t^3 & 0 \\ 0 & t^3 \\ t & t^2 \end{pmatrix} \end{array}$$

$$\begin{array}{c} \\ cd \\ bc \\ ab \\ ad + cd + t \cdot bc \\ ac \end{array} \begin{array}{c} \begin{array}{cc} abc & acd \end{array} \\ \begin{pmatrix} 0 & 0 \\ t^3 & t^4 \\ t^3 & 0 \\ 0 & t^3 \\ t & t^2 \end{pmatrix} \end{array}$$

$$\begin{array}{l} \\ cd \\ bc \\ ab \\ ad + cd + t \cdot bc \\ ac \end{array} \begin{array}{c} \begin{array}{cc} abc & acd \end{array} \\ \begin{pmatrix} 0 & 0 \\ t^3 & t^4 \\ t^3 & 0 \\ 0 & t^3 \\ t & t^2 \end{pmatrix} \end{array}$$

$$\begin{array}{l} \\ cd \\ bc \\ ab \\ ad + cd + t \cdot bc \\ ac + t^2 \cdot bc \end{array} \begin{array}{c} \begin{array}{cc} abc & acd \end{array} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ t^3 & 0 \\ 0 & t^3 \\ t & t^2 \end{pmatrix} \end{array}$$

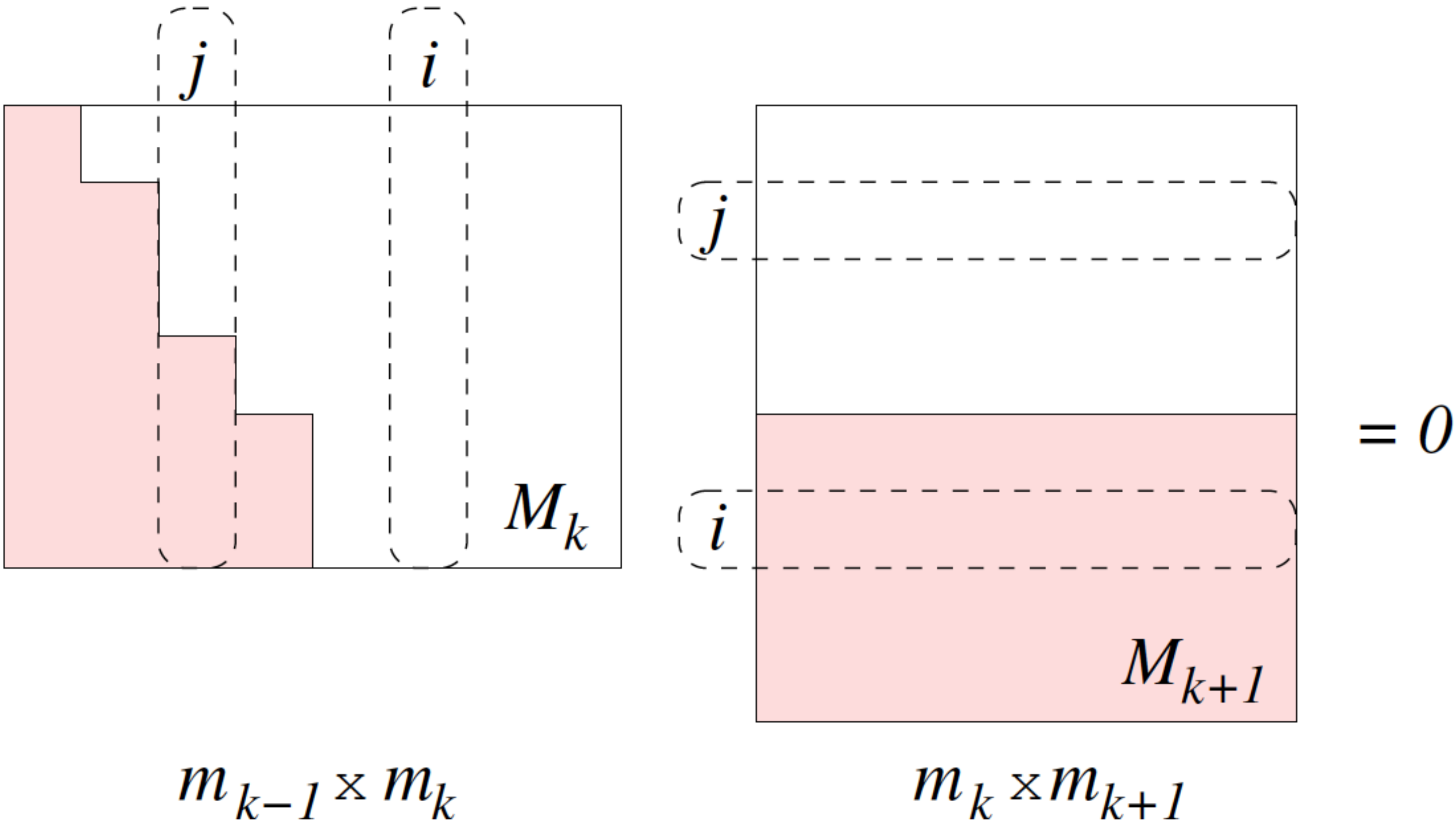
$$
\begin{array}{l}
 \\
cd \\
bc \\
ab \\
ad + cd + t \cdot bc \\
ac + t^2 \cdot bc
\end{array}
\begin{array}{c}
\begin{array}{cc} abc & acd \end{array} \\
\begin{pmatrix}
0 & 0 \\
0 & 0 \\
t^3 & 0 \\
0 & t^3 \\
t & t^2
\end{pmatrix}
\end{array}
$$

$$
\begin{array}{l}
 \\
cd \\
bc \\
ab \\
ad + cd + t \cdot bc + t \cdot ab \\
ac + t^2 \cdot bc
\end{array}
\begin{array}{c}
\begin{array}{cc} abc & acd \end{array} \\
\begin{pmatrix}
0 & 0 \\
0 & 0 \\
t^3 & t^4 \\
0 & t^3 \\
t & t^2
\end{pmatrix}
\end{array}
$$

$$
\begin{array}{l} \\ cd \\ bc \\ ab \\ ad + cd + t\cdot bc + t\cdot ab \\ ac + t^2\cdot bc \end{array}
\begin{array}{c} \begin{array}{cc} abc & acd \end{array} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ t^3 & t^4 \\ 0 & t^3 \\ t & t^2 \end{pmatrix} \end{array}
$$

$$
\begin{array}{l} \\ cd \\ bc \\ ab \\ ad + cd + t\cdot bc + t\cdot ab \\ ac + t^2\cdot bc + t^2\cdot ab \end{array}
\begin{array}{c} \begin{array}{cc} abc & acd \end{array} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & t^3 \\ t & t^2 \end{pmatrix} \end{array}
$$

$C_{k+1} \xrightarrow{\partial_{k+1}} C_k \xrightarrow{\partial_k} C_{k-1}$, m_k = # of k-simplices



http://link.springer.com/article/10.1007%2Fs00454-004-1146-y

$$\begin{pmatrix} t & 0 & 0 & 0 & 0 \\ t & 1 & 0 & 0 & 0 \\ 0 & t & t & 0 & 0 \\ 0 & 0 & t & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & t^3 \\ t & t^2 \end{pmatrix} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$
\begin{array}{c} \\ d \\ c \\ b \\ a \end{array}
\begin{array}{c}
\begin{array}{ccccc} cd & bc & ab & ad + cd + t \cdot bc + t \cdot ab & ac + t^2 \cdot bc + t^2 \cdot ab \end{array} \\
\begin{pmatrix} t & 0 & 0 & 0 & 0 \\ t & 1 & 0 & 0 & 0 \\ 0 & t & t & 0 & 0 \\ 0 & 0 & t & 0 & 0 \end{pmatrix}
\end{array}
$$

$$
\begin{array}{l} \\ cd \\ bc \\ ab \\ ad + cd + t \cdot bc + t \cdot ab \\ ac + t^2 \cdot bc + t^2 \cdot ab \end{array}
\begin{array}{c}
\begin{array}{cc} abc & acd \end{array} \\
\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & t^3 \\ t & t^2 \end{pmatrix}
\end{array}
$$

Short method for determining column space of M_2

$$
\begin{array}{c} \\ cd \\ bc \\ ab \\ ad \\ ac \end{array}
\begin{array}{c} \begin{array}{cc} abc & acd \end{array} \\ \begin{pmatrix} 0 & t^3 \\ t^3 & 0 \\ t^3 & 0 \\ 0 & t^3 \\ t & t^2 \end{pmatrix} \end{array}
$$

$$
\begin{array}{l} \\ cd \\ bc \\ ab \\ ad + cd + t \cdot bc + t \cdot ab \\ ac + t^2 \cdot bc + t^2 \cdot ab \end{array}
\begin{array}{c} \begin{array}{cc} abc & acd \end{array} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & t^3 \\ t & t^2 \end{pmatrix} \end{array}
$$

$$C_2 \xrightarrow{\partial_1} C_1 \xrightarrow{\partial_1} C_0$$

$H_1 = Z_1/B_1 = (\text{kernel of } \partial_1)/\ (\text{image of } \partial_2)$

$$= \frac{\text{null space of } M_1}{\text{column space of } M_2}$$

$$= \ < ad + cd + t(bc) + t(ab),\ \ ac + t^2bc + t^2ab \ \ : \ \ ??????? >$$

Let $z_1 = ad + cd + t(bc) + t(ab)$, $z_2 = ac + t^2bc + t^2ab$

$$H_1 = Z_1/B_1 = \ <z_1, z_2 \ : \ ?? >$$

$$
\begin{array}{l} \\ cd \\ bc \\ ab \\ ad + cd + t \cdot bc + t \cdot ab \\ ac + t^2 \cdot bc + t^2 \cdot ab \end{array}
\begin{array}{c} \begin{array}{cc} abc & acd \end{array} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & t^3 \\ t & t^2 \end{pmatrix} \end{array}
$$

image of ∂_2 =

kernel of ∂_2 =

$$
\begin{array}{l}
cd \\
bc \\
ab \\
ad + cd + t \cdot bc + t \cdot ab \\
ac + t^2 \cdot bc + t^2 \cdot ab
\end{array}
\begin{array}{c}
\begin{array}{cc} abc & acd \end{array} \\
\begin{pmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & t^3 \\
t & t^2
\end{pmatrix}
\end{array}
$$

image of $\partial_2 = < t\, z_2,\; t^3 z_1 + t^2 z_2 >$

kernel of $\partial_2 = 0$

$$C_2 \xrightarrow{\partial_2} C_1 \xrightarrow{\partial_1} C_0$$

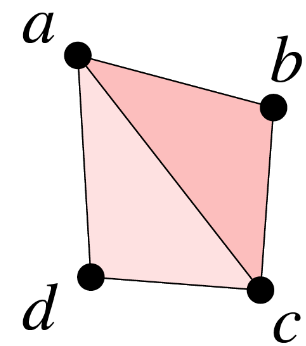
$H_1 = Z_1/B_1 = (\text{kernel of } \partial_1)/(\text{image of } \partial_2)$

$$= \frac{\text{null space of } M_1}{\text{column space of } M_2}$$

$$= \langle z_1, z_2 \;:\; t\, z_2,\; t^3 z_1 + t^2 z_2 \rangle$$

where

$z_1 = ad + cd + t(bc) + t(ab), \quad z_2 = ac + t^2bc + t^2ab$



$$C_3 \xrightarrow{\partial_3} C_2 \xrightarrow{\partial_2} C_1$$

$H_2 = Z_2/B_2 =$ (kernel of ∂_2)/ (image of ∂_3)

$$= \frac{\text{null space of } M_2}{\text{column space of } M_3}$$

$$= 0$$

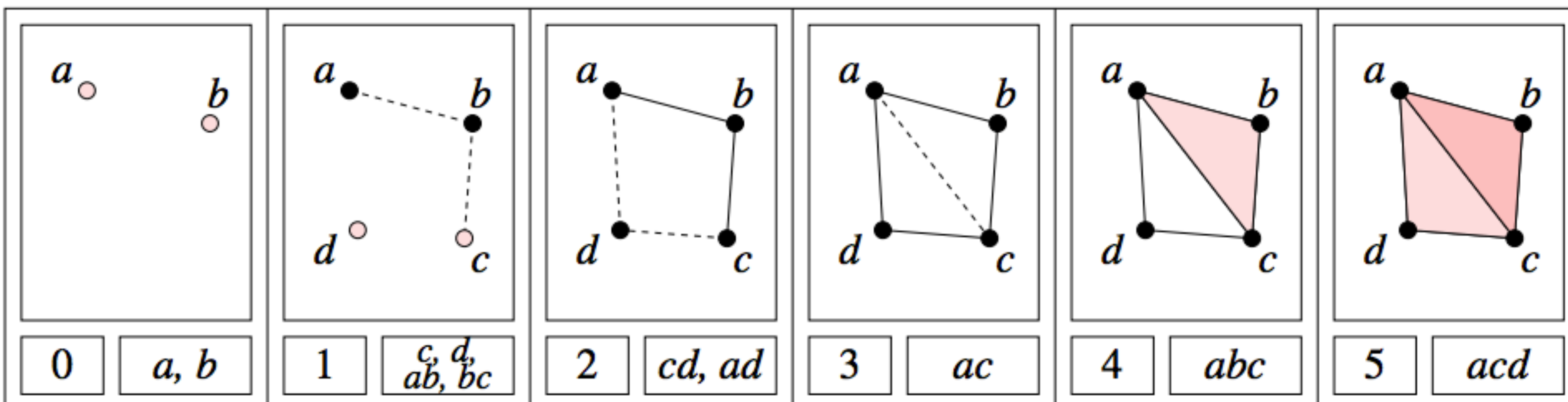
$< z_1, z_2 \; : \; t\, z_2, \; t^3 z_1 + t^2 z_2 \; >$ where

$z_1 = ad + cd + t(bc) + t(ab), \quad z_2 = ac + t^2 bc + t^2 ab$

$$H_1^{i,p} = Z_1^i / (B_1^{i+p} \cap Z_1^i)$$

Note deg $z_1 = 2$, deg $z_2 = 3$

$$H_1^{i,p} = Z_1^i / (B_1^{i+p} \cap Z_1^i) = 0 \text{ for } i < 2$$



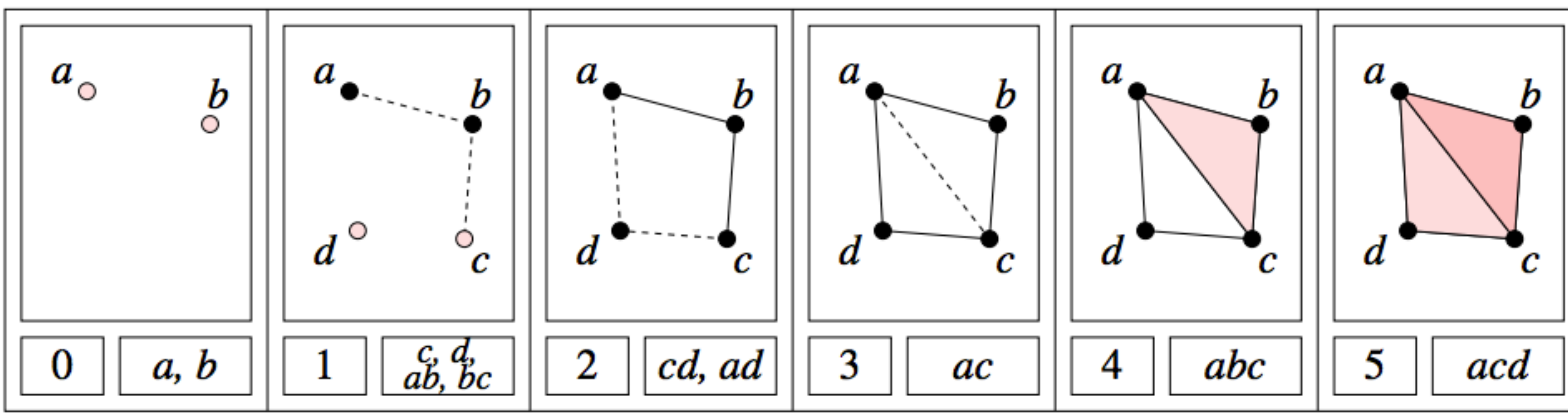
$\langle z_1, z_2 : tz_2, t^3z_1 + t^2z_2 \rangle$ where

$z_1 = ad + cd + t(bc) + t(ab), \quad z_2 = ac + t^2bc + t^2ab$

$$H_1^{i,p} = Z_1^i / (B_1^{i+p} \cap Z_1^i)$$

$\deg z_1 = 2$, $\deg z_2 = 3$, $\deg tz_2 = 4$, $\deg t^3z_1 + t^2z_2 = 5$

$$H_1^{2,p} = Z_1^2 / (B_1^{2+p} \cap Z_1^2) = \mathbf{Z}/2\mathbf{Z} \text{ for } p =$$



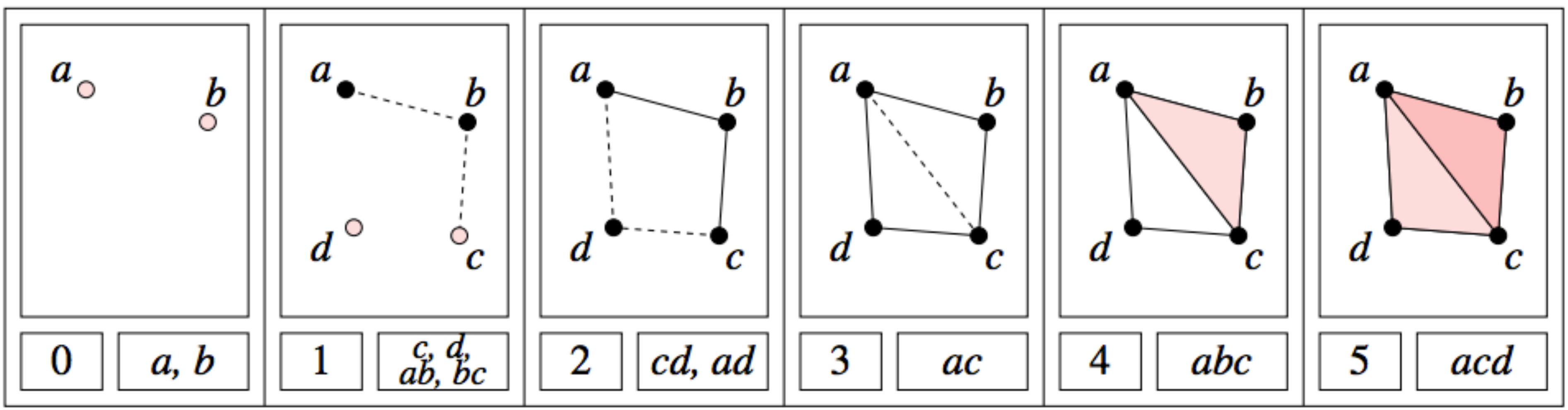
$<z_1, z_2 : tz_2, t^3 z_1 + t^2 z_2>$ where

$z_1 = ad + cd + t(bc) + t(ab), \quad z_2 = ac + t^2 bc + t^2 ab$

$$H_1^{i,p} = Z_1^i / (B_1^{i+p} \cap Z_1^i)$$

$\deg z_1 = 2$, $\deg z_2 = 3$, $\deg tz_2 = 4$, $\deg t^3 z_1 + t^2 z_2 = 5$

$$H_1^{2,p} = Z_1^2 / (B_1^{2+p} \cap Z_1^2) = \mathbf{Z}/2\mathbf{Z} \text{ for } p = 0, 1, 2$$



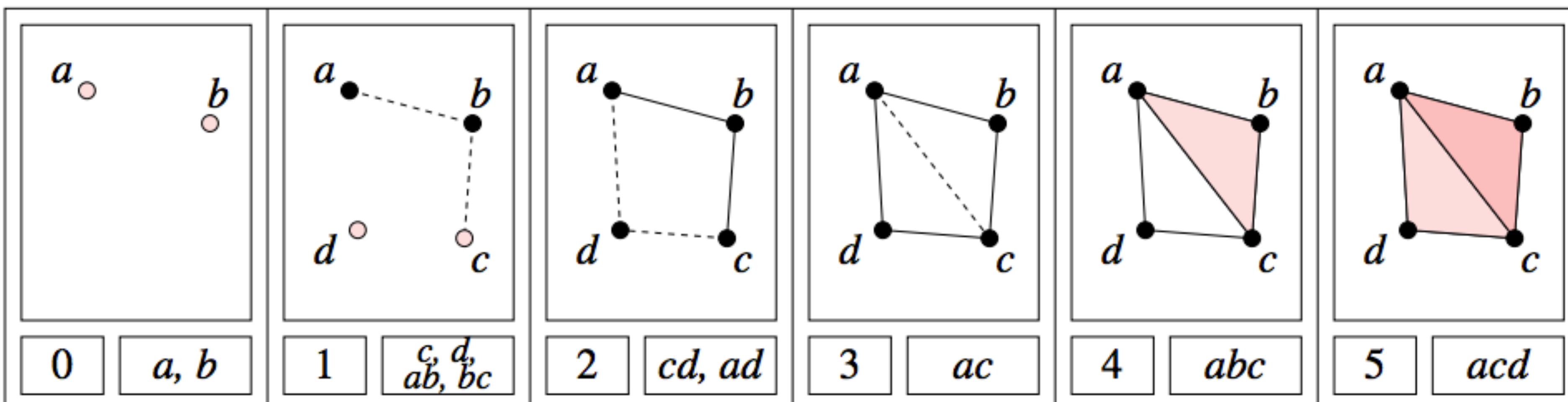
$\langle z_1, z_2 \;:\; tz_2,\; t^3z_1 + t^2z_2 \rangle$ where

$z_1 = ad + cd + t(bc) + t(ab)$, $z_2 = ac + t^2bc + t^2ab$

$$H_1^{i,p} = Z_1^i / (B_1^{i+p} \cap Z_1^i)$$

$\deg z_1 = 2$, $\deg z_2 = 3$, $\deg tz_2 = 4$, $\deg t^3z_1 + t^2z_2 = 5$

$$H_1^{2,p} = Z_1^2 / (B_1^{2+p} \cap Z_1^2) = 0 \text{ for } p =$$



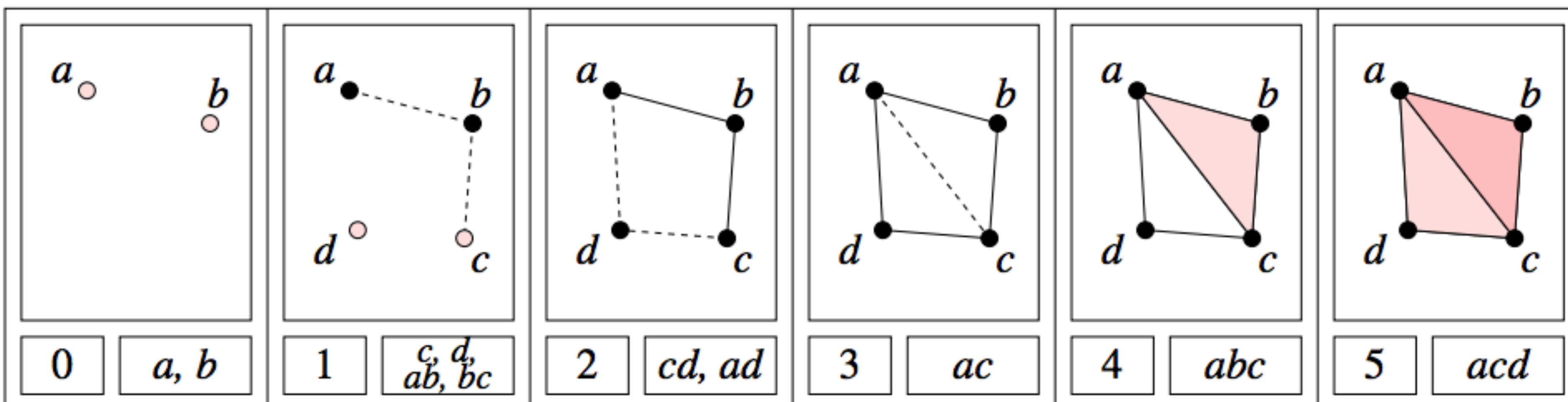
$< z_1, z_2 \;:\; tz_2,\; t^3z_1 + t^2z_2 >$ where

$z_1 = ad + cd + t(bc) + t(ab), \quad z_2 = ac + t^2bc + t^2ab$

$$H_1^{i,p} = Z_1^i / (B_1^{i+p} \cap Z_1^i)$$

$\deg z_1 = 2$, $\deg z_2 = 3$, $\deg tz_2 = 4$, $\deg t^3z_1 + t^2z_2 = 5$

$$H_1^{2,p} = Z_1^2 / (B_1^{2+p} \cap Z_1^2) = 0 \text{ for } p = 3$$



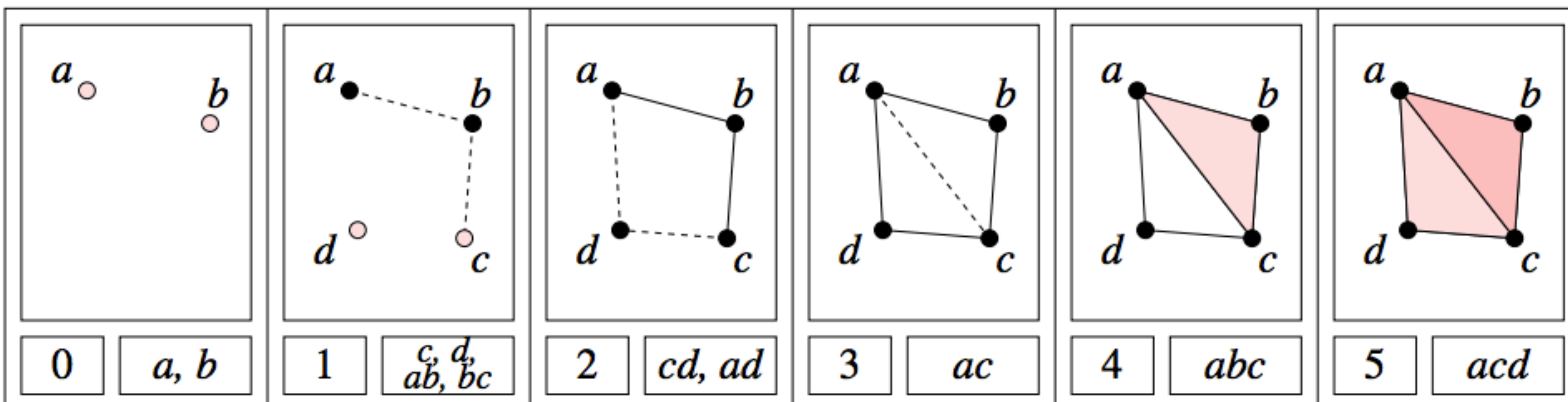
$< z_1, z_2 \;:\; tz_2,\; t^3z_1 + t^2z_2 >$ where

$z_1 = ad + cd + t(bc) + t(ab)$, $z_2 = ac + t^2bc + t^2ab$

$$H_1^{i,\,p} = Z_1^i / (B_1^{i+p} \cap Z_1^i)$$

$\deg z_1 = 2$, $\deg z_2 = 3$, $\deg tz_2 = 4$, $\deg t^3z_1 + t^2z_2 = 5$

$$H_1^{3,\,p} = Z_1^3 / (B_1^{3+p} \cap Z_1^3) = \;???$$



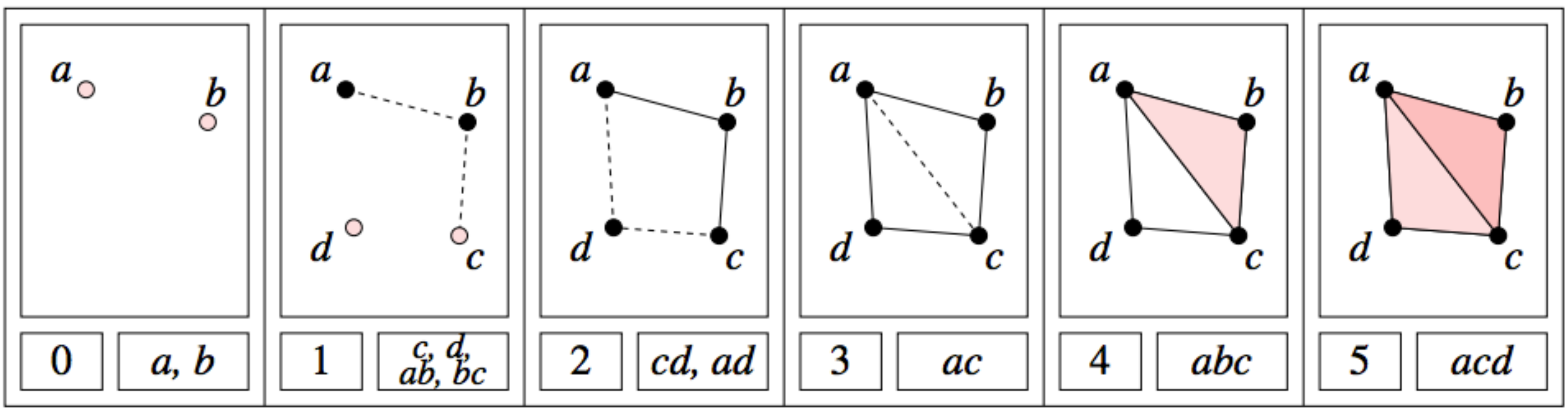
$< z_1, z_2 \; : \; tz_2, \; t^3 z_1 + t^2 z_2 >$ where

$z_1 = ad + cd + t(bc) + t(ab), \quad z_2 = ac + t^2 bc + t^2 ab$

$$H_1^{i,\,p} = Z_1^i / (B_1^{i+p} \cap Z_1^i)$$

$\deg z_1 = 2$, $\deg z_2 = 3$, $\deg tz_2 = 4$, $\deg t^3 z_1 + t^2 z_2 = 5$

$$H_1^{4,\,0} = Z_1^4 / (B_1^{4+0} \cap Z_1^4) =$$



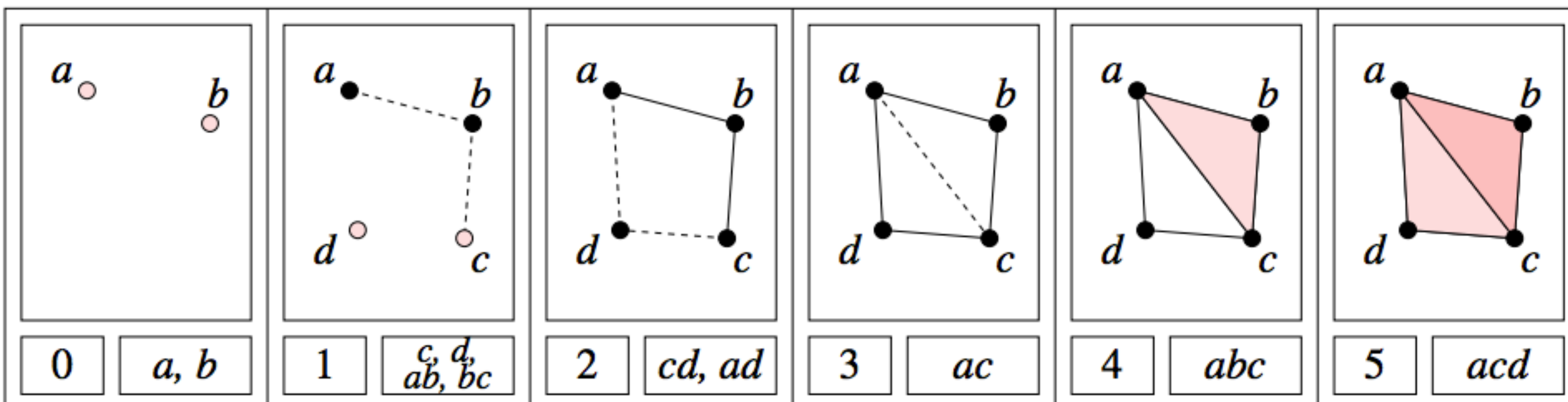
$\langle z_1, z_2 \;:\; tz_2,\; t^3z_1 + t^2z_2 \rangle$ where

$z_1 = ad + cd + t(bc) + t(ab)$, $z_2 = ac + t^2bc + t^2ab$

$$H_1^{i,p} = Z_1^i / (B_1^{i+p} \cap Z_1^i)$$

$\deg z_1 = 2$, $\deg z_2 = 3$, $\deg tz_2 = 4$, $\deg t^3z_1 + t^2z_2 = 5$

$$H_1^{4,1} = Z_1^4 / (B_1^{4+1} \cap Z_1^4) =$$



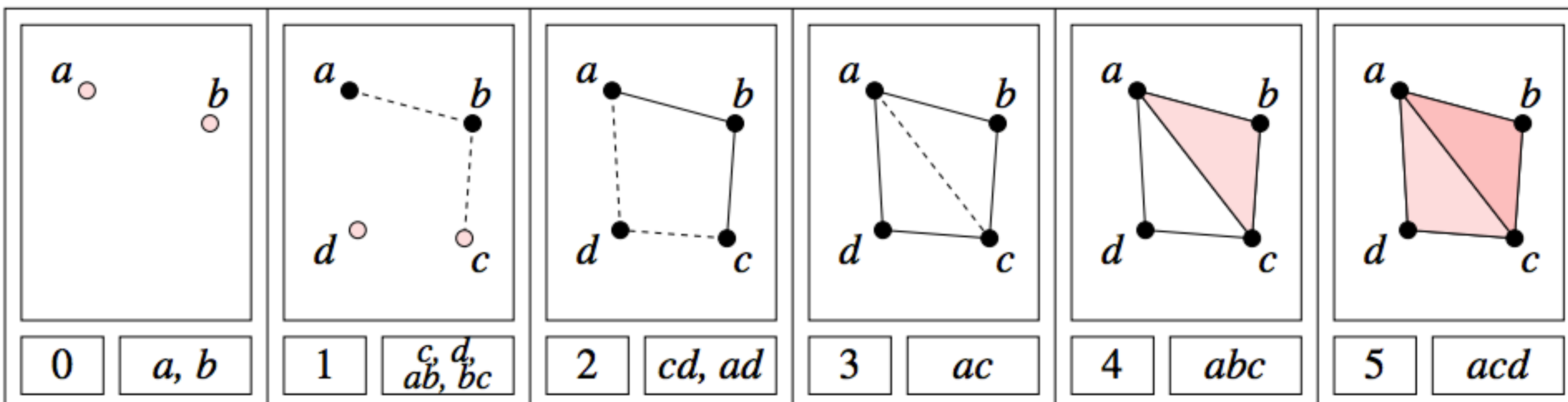
$< z_1, z_2 \; : \; tz_2, \; t^3z_1 + t^2z_2 \; >$ where

$z_1 = ad + cd + t(bc) + t(ab), \quad z_2 = ac + t^2bc + t^2ab$

$$H_1^{i,p} = Z_1^i / (B_1^{i+p} \cap Z_1^i)$$

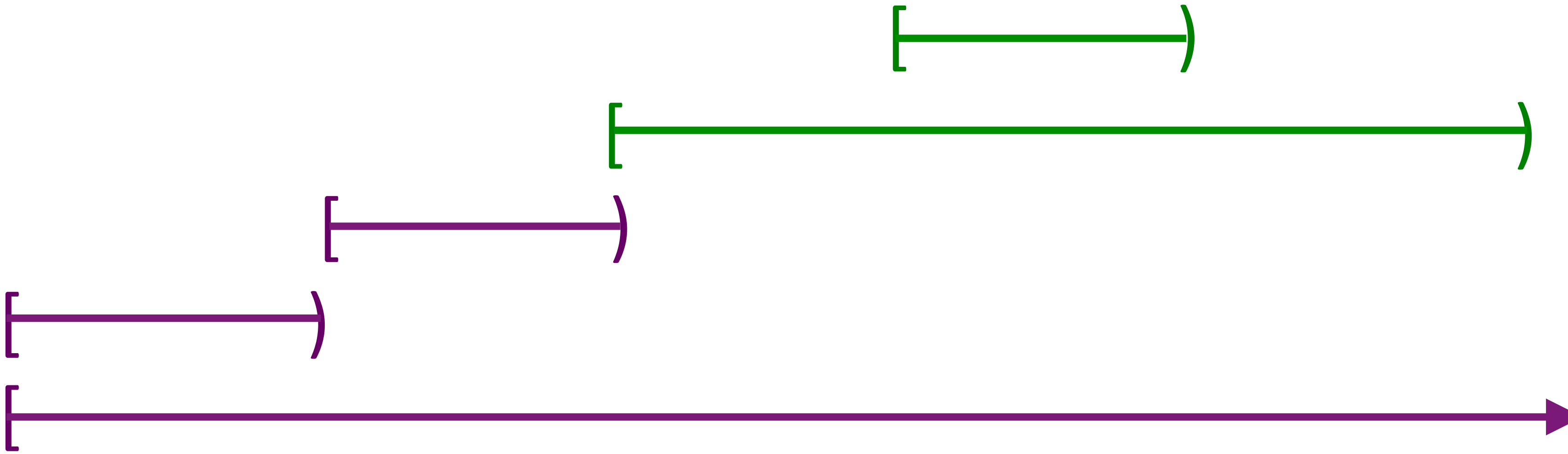
$\deg z_1 = 2$, $\deg z_2 = 3$, $\deg tz_2 = 4$, $\deg t^3z_1 + t^2z_2 = 5$

$$H_1^{5,0} = Z_1^5 / (B_1^{5+0} \cap Z_1^5) =$$

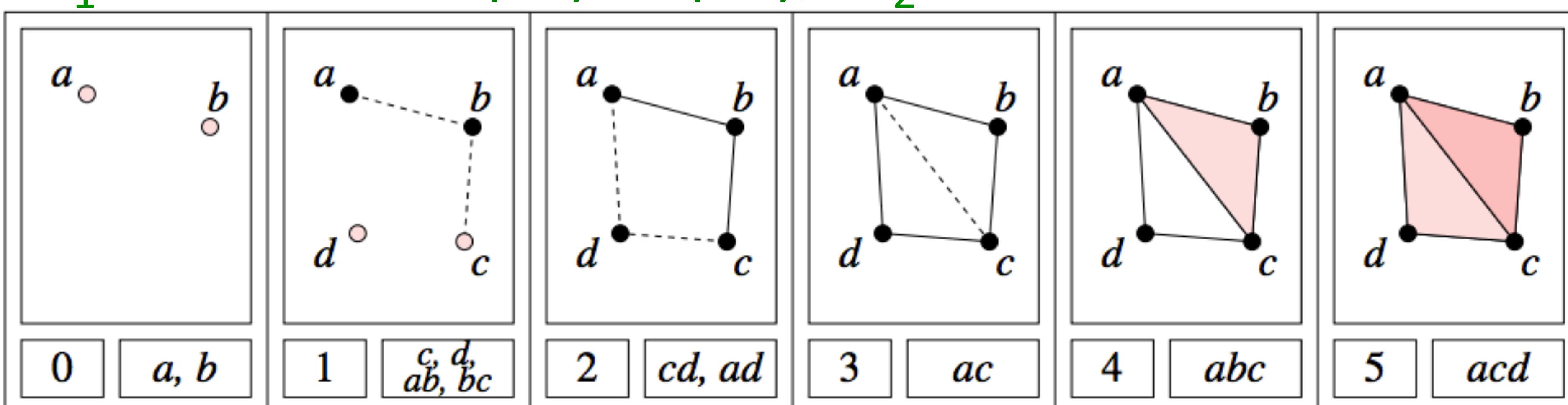


$$H_0 = \langle a, b, c, d : tc + td,\ tb + c,\ ta + tb \rangle$$

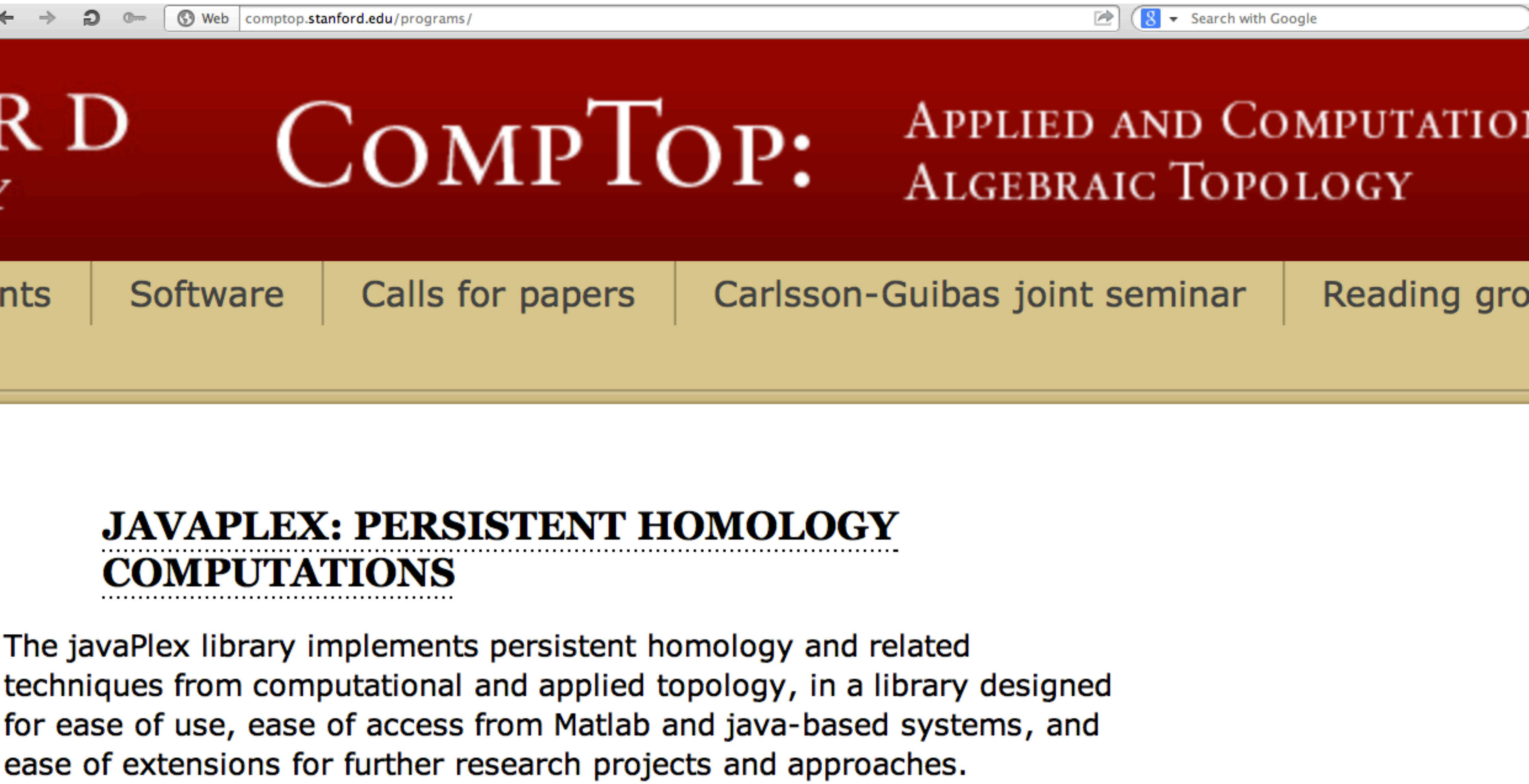
$$H_1 = \langle z_1, z_2 : t z_2,\ t^3 z_1 + t^2 z_2 \rangle$$



$$z_1 = ad + cd + t(bc) + t(ab), \quad z_2 = ac + t^2 bc + t^2 ab$$



http://comptop.stanford.edu/programs/



Tausz, Andrew; Vejdemo-Johansson, Mikael; Adams, Henry

Download: http://code.google.com/p/javaplex/

Welcome to javaPlex

The javaPlex library implements persistent homology and related techniques from computational and applied topology, in a library designed for ease of use, ease of access from Matlab and java-based systems, and ease of extensions for further research projects and approaches.

javaPlex is mainly developed by the Computational Topology workgroup at Stanford University, and is based on previous similar packages from the same group.

For persistent homology and its capabilities, we recommend the survey article Topology and Data by Gunnar Carlsson.

How to get started?

- Start playing around with the latest matlab examples
- Read the tutorial
- Take a look at the wiki overview

Download: http://code.google.com/p/javaplex/

eatured

Downloads

- javaplex-4.1.0.jar
- javaplex-doc-4.1.0.tar.gz
- javaplex-doc-4.1.0.zip
- javaplex-processing-demo-4.1.0.zip
- javaplex-processing-lib-4.1.0.zip
- javaplex-src-4.1.0.tar.gz
- javaplex-src-4.1.0.zip
- javaplex-tutorial-4.1.0.pdf
- matlab-examples-4.1.0.tar.gz
- matlab-examples-4.1.0.zip
- Show all »

How to get s

- Start playing arour
- Read the tutorial
- Take a look at the
- Download the late

For more in

- Read the wiki over
- Read about the ar
- Look at the javadc

Some useful linux/mac/matlab commands:

tar -xvvf foo.tar	extract foo.tar
tar -xvvzf foo.tar.gz	extract gzipped foo.tar.gz
cd Directory	change directory
cd ../	go up one directory
cd	go to main directory
pwd	print working directory
ls	list directory content
ls –lrt	list long format in reverse order time

JAVAPLEX TUTORIAL

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```
>> version -java
ans = Java 1.6.0_17-b04 *************
```

javaPlex requires version number 1.5 or higher.

```
>> cd AT/matlab_examples
```

```
>> load_javaplex
```

Confirm that javaPlex is working properly:

```
>> api.Plex4.createExplicitSimplexStream()
ans =
edu.stanford.math.plex4.streams.impl.ExplicitSimplexStream@513fd4
```

create an empty explicit simplex stream:

```
>> stream =
api.Plex4.createExplicitSimplexStream();
```

add simplicies:

```
>> stream.addVertex(0);
>> stream.addVertex(1);
>> stream.addVertex(2);
>> stream.addElement([0, 1]);
>> stream.addElement([0, 2]);
>> stream.addElement([1, 2]);
>> stream.finalizeStream();
```

Create filtered complex:

```
>> stream = api.Plex4.createExplicitSimplexStream();
>> stream.addVertex(1, 0);
>> stream.addVertex(2, 0);
>> stream.addVertex(3, 0);
>> stream.addVertex(4, 0);
>> stream.addVertex(5, 1);
>> stream.addElement([1, 2], 0);
>> stream.addElement([2, 3], 0);
>> stream.addElement([3, 4], 0);
>> stream.addElement([4, 1], 0);
>> stream.addElement([3, 5], 2);
>> stream.addElement([4, 5], 3);
>> stream.addElement([3, 4, 5], 7);
>> stream.finalizeStream();
```

Determine if you have created a simplicial complex:

```
>> stream.validateVerbose()
ans = 1
>> stream.addElement([1, 4, 5], 0);
>> stream.validateVerbose()
Filtration index of face [4,5] exceeds that of element
[1,4,5] (3 > 0)
Stream does not contain face [1,5] of element [1,4,5]
ans = 0
```

Create 2-dimensional sphere, S^2

```
 >> dimension = 2;
>> stream = api.Plex4.createExplicitSimplexStream();
>> stream.addElement(0:(dimension + 1));
>> stream.ensureAllFaces();
>> stream.removeElementIfPresent(0:(dimension + 1));
>> stream.finalizeStream();
```

Determine H_i for $i < 3$ with Z_2 coefficients:

```
>> persistence =
api.Plex4.getModularSimplicialAlgorithm(3, 2);
>> intervals = persistence.computeIntervals(stream)
intervals =

Dimension: 1
[3.0, 7.0)
[0.0, infinity)
Dimension: 0
[1.0, 2.0)
[0.0, infinity)
```

compute a representative cycle for each barcode:

```
>> intervals =
persistence.computeAnnotatedIntervals(stream)

intervals =

Dimension: 1
[3.0, 7.0): [4,5] + [3,4] + -[3,5]
[0.0, infinity): [1,4] + [2,3] + [1,2] + [3,4]
Dimension: 0
[1.0, 2.0): -[3] + [5]
[0.0, infinity): [1]
```

To enter a CSV file into Matlab, you can use the following command:

```
>> M = csvread('filename',row,col);
```

reads data from the file starting at the specified row and column. The row and column arguments are zero based, so that row = 0 and col = 0 specify the first value in the file.

From:
http://www.mathworks.com/help/matlab/ref/csvread.html