Optional Lecture: A terse introduction to simplicial complexes

in a series of preparatory lectures for the Fall 2013 online course MATH:7450 (22M:305) Topics in Topology: Scientific and Engineering Applications of Algebraic Topology

Target Audience: Anyone interested in **topological data analysis** including graduate students, faculty, industrial researchers in bioinformatics, biology, computer science, cosmology, engineering, imaging, mathematics, neurology, physics, statistics, etc.

Isabel K. Darcy

Mathematics Department/Applied Mathematical & Computational Sciences

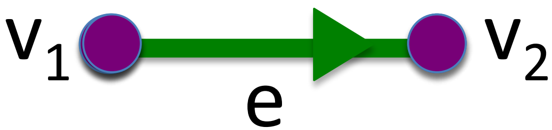
University of Iowa

http://www.math.uiowa.edu/~idarcy/AppliedTopology.html

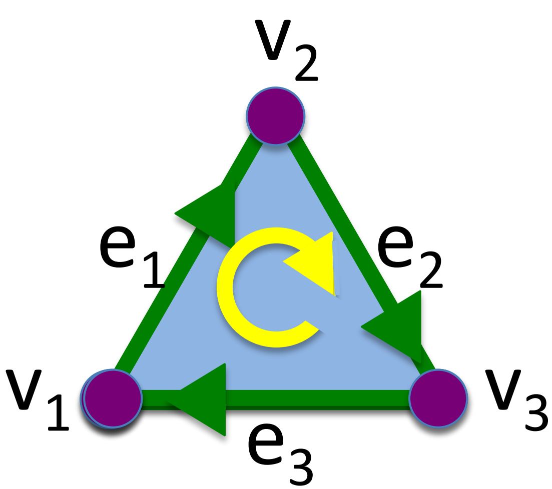
Building blocks for oriented simplicial complex

0-simplex = vertex = v

1-simplex = oriented edge = (v_1, v_2)



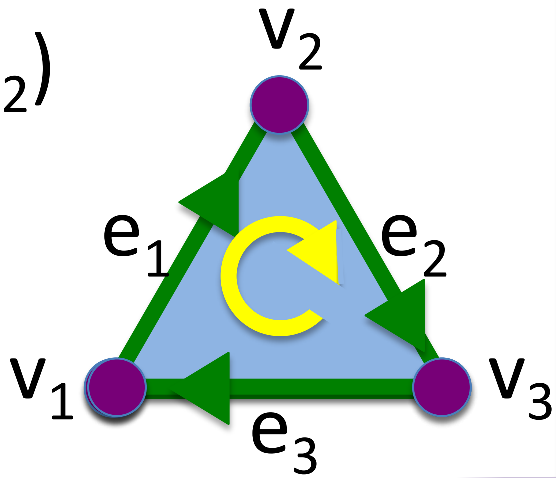
2-simplex = oriented face = (v_1, v_2, v_3)



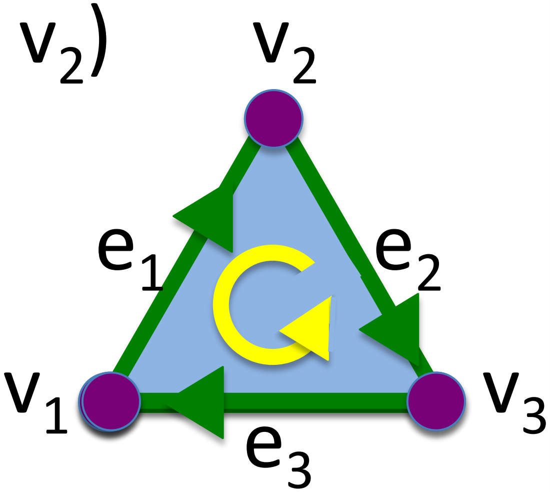
Building blocks for oriented simplicial complex

2-simplex = oriented face =

$f = (v_1, v_2, v_3) = (v_2, v_3, v_1) = (v_3, v_1, v_2)$



$-f = (v_2, v_1, v_3) = (v_3, v_2, v_1) = (v_1, v_3, v_2)$

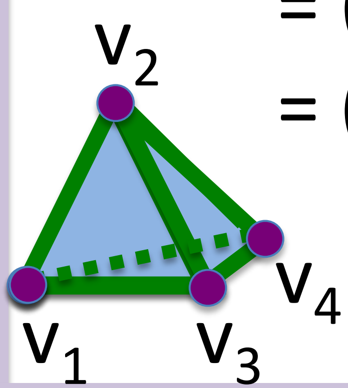


Building blocks for oriented simplicial complex

3-simplex =

$$\sigma = (v_1, v_2, v_3, v_4) = (v_2, v_3, v_1, v_4) = (v_3, v_1, v_{2,} v_4)$$
$$= (v_2, v_1, v_4, v_3) = (v_3, v_2, v_4, v_1) = (v_1, v_3, v_4, v_2)$$
$$= (v_4, v_2, v_1, v_3) = (v_4, v_3, v_2, v_1) = (v_4, v_1, v_3, v_2)$$
$$= (v_1, v_4, v_2, v_3) = (v_2, v_4, v_3, v_1) = (v_3, v_4, v_1, v_2)$$

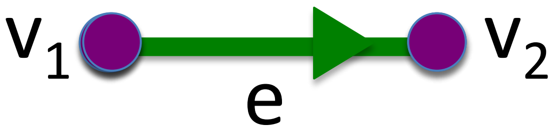
$$-\sigma = (v_2, v_1, v_3, v_4) = (v_3, v_2, v_1, v_4) = (v_1, v_3, v_2, v_4)$$
$$= (v_2, v_4, v_1, v_3) = (v_3, v_4, v_2, v_1) = (v_1, v_4, v_3, v_2)$$
$$= (v_1, v_2, v_4, v_3) = (v_2, v_3, v_4, v_1) = (v_3, v_1, v_4, v_2)$$
$$= (v_4, v_1, v_2, v_3) = (v_4, v_2, v_3, v_1) = (v_4, v_3, v_1, v_2)$$



Building blocks for oriented simplicial complex

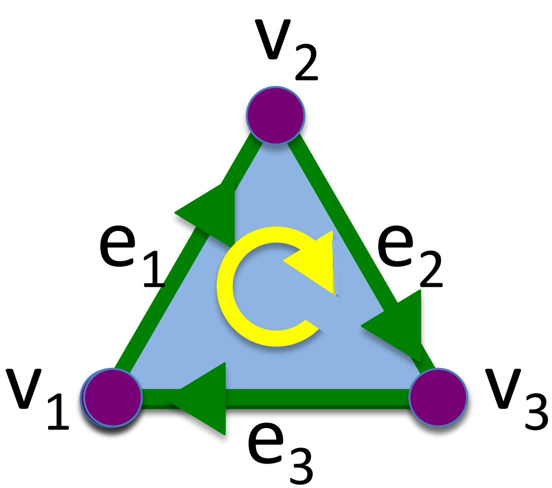
0-simplex = vertex = v

1-simplex = oriented edge = (v_1, v_2)



Note that the boundary of this edge is $v_2 - v_1$

2-simplex = oriented face = (v_1, v_2, v_3)



Note that the boundary of this face is the cycle

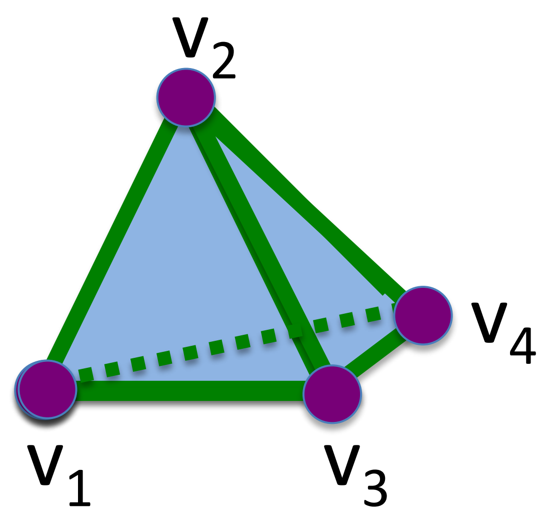
$$e_1 + e_2 + e_3$$

$$= (v_1, v_2) + (v_2, v_3) - (v_1, v_3)$$

$$= (v_1, v_2) - (v_1, v_3) + (v_2, v_3)$$

Building blocks for oriented simplicial complex

3-simplex = (v_1, v_2, v_3, v_4) = tetrahedron



boundary of (v_1, v_2, v_3, v_4) =

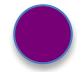
$$-(v_1, v_2, v_3) + (v_1, v_2, v_4) - (v_1, v_3, v_4) + (v_2, v_3, v_4)$$

---

n-simplex = $(v_1, v_2, \ldots, v_{n+1})$

Building blocks for an unoriented simplicial complex using $\mathbf{Z}_2$ coefficients

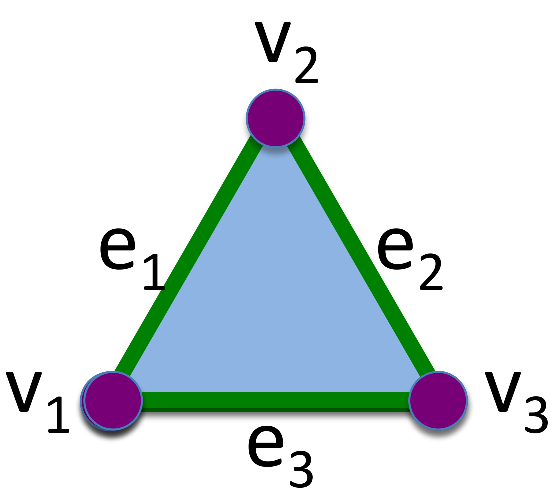
0-simplex = vertex = v



1-simplex = edge = $\{v_1, v_2\}$

Note that the boundary of this edge is $v_2 + v_1$

2-simplex = face = $\{v_1, v_2, v_3\}$



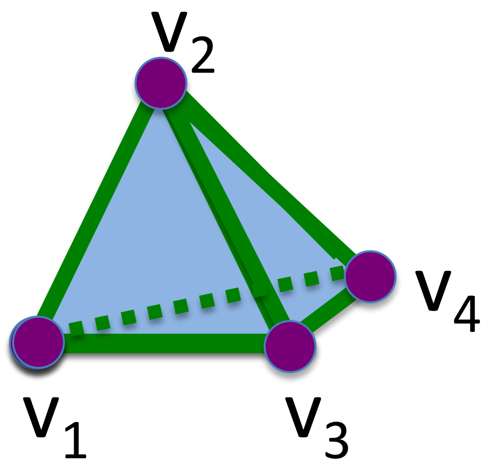
Note that the boundary of this face is the cycle

$$e_1 + e_2 + e_3$$

$$= \{v_1, v_2\} + \{v_2, v_3\} + \{v_1, v_3\}$$

Building blocks for a simplicial complex using $\mathbf{Z_2}$ coefficients

3-simplex = $\{v_1, v_2, v_3, v_4\}$ = tetrahedron



boundary of $\{v_1, v_2, v_3, v_4\}$ =

$\{v_1, v_2, v_3\} + \{v_1, v_2, v_4\} + \{v_1, v_3, v_4\} + \{v_2, v_3, v_4\}$

---

n-simplex = $\{v_1, v_2, \ldots, v_{n+1}\}$