Optional Lecture: A terse introduction to simplicial complexes

in a series of preparatory lectures for the Fall 2013 online course MATH:7450 (22M:305) Topics in Topology: Scientific and Engineering Applications of Algebraic Topology

Target Audience: Anyone interested in topological data analysis including graduate students, faculty, industrial researchers in bioinformatics, biology, computer science, cosmology, engineering, imaging, mathematics, neurology, physics, statistics, etc.

Isabel K. Darcy
Mathematics Department/Applied Mathematical & Computational Sciences
University of Iowa

http://www.math.uiowa.edu/~idarcy/AppliedTopology.html
Building blocks for oriented simplicial complex

0-simplex = vertex = \( v \)

1-simplex = oriented edge = \((v_1, v_2)\)

\[ \begin{align*}
\text{v}_1 & \quad \text{e} \quad \text{v}_2 \\
\end{align*} \]

2-simplex = oriented face = \((v_1, v_2, v_3)\)

\[ \begin{align*}
\text{v}_1 & \quad \text{e}_1 \quad \text{e}_2 \quad \text{e}_3 \\
\end{align*} \]
Building blocks for oriented simplicial complex

2-simplex = oriented face = 

\[ f = (v_1, v_2, v_3) = (v_2, v_3, v_1) = (v_3, v_1, v_2) \]

\[ -f = (v_2, v_1, v_3) = (v_3, v_2, v_1) = (v_1, v_3, v_2) \]
Building blocks for oriented simplicial complex

3-simplex =

\[ \sigma = (v_1, v_2, v_3, v_4) = (v_2, v_3, v_1, v_4) = (v_3, v_1, v_2, v_4) = (v_4, v_2, v_1, v_3) = (v_4, v_3, v_2, v_1) = (v_4, v_1, v_3, v_2) = (v_1, v_4, v_2, v_3) = (v_2, v_4, v_3, v_1) = (v_3, v_4, v_1, v_2) \]

\[-\sigma = (v_2, v_1, v_3, v_4) = (v_3, v_2, v_1, v_4) = (v_1, v_3, v_2, v_4) = (v_2, v_4, v_1, v_3) = (v_3, v_4, v_2, v_1) = (v_1, v_4, v_3, v_2) = (v_1, v_2, v_4, v_3) = (v_2, v_3, v_4, v_1) = (v_3, v_1, v_4, v_2) = (v_4, v_1, v_2, v_3) = (v_4, v_2, v_3, v_1) = (v_4, v_3, v_1, v_2) \]
Building blocks for oriented simplicial complex

0-simplex = vertex = $v$

1-simplex = oriented edge = $(v_1, v_2)$

Note that the boundary of this edge is $v_2 - v_1$

2-simplex = oriented face = $(v_1, v_2, v_3)$

Note that the boundary of this face is the cycle

\[ e_1 + e_2 + e_3 = (v_1, v_2) + (v_2, v_3) - (v_1, v_3) \]
\[ = (v_1, v_2) - (v_1, v_3) + (v_2, v_3) \]
Building blocks for oriented simplicial complex

3-simplex = \((v_1, v_2, v_3, v_4)\) = tetrahedron

boundary of \((v_1, v_2, v_3, v_4)\) =

\[- (v_1, v_2, v_3) + (v_1, v_2, v_4) - (v_1, v_3, v_4) + (v_2, v_3, v_4)\]

n-simplex = \((v_1, v_2, ..., v_{n+1})\)
Building blocks for an unoriented simplicial complex using $\mathbb{Z}_2$ coefficients

**0-simplex = vertex = $v$**

**1-simplex = edge = $\{v_1, v_2\}$**

Note that the boundary of this edge is $v_2 + v_1$

**2-simplex = face = $\{v_1, v_2, v_3\}$**

Note that the boundary of this face is the cycle:

$$e_1 + e_2 + e_3 = \{v_1, v_2\} + \{v_2, v_3\} + \{v_1, v_3\}$$
Building blocks for a simplicial complex using $\mathbb{Z}_2$ coefficients

3-simplex = $\{v_1, v_2, v_3, v_4\}$ = tetrahedron

boundary of $\{v_1, v_2, v_3, v_4\}$ =
$\{v_1, v_2, v_3\} + \{v_1, v_2, v_4\} + \{v_1, v_3, v_4\} + \{v_2, v_3, v_4\}$

$n$-simplex = $\{v_1, v_2, ..., v_{n+1}\}$