Lecture 3: Modular Arithmetic

of a series of preparatory lectures for the Fall 2013 online course MATH:7450 (22M:305) Topics in Topology: Scientific and Engineering Applications of Algebraic Topology

Target Audience: Anyone interested in **topological data analysis** including graduate students, faculty, industrial researchers in bioinformatics, biology, computer science, cosmology, engineering, imaging, mathematics, neurology, physics, statistics, etc.

Isabel K. Darcy

Mathematics Department/Applied Mathematical & Computational Sciences

University of Iowa

http://www.math.uiowa.edu/~idarcy/AppliedTopology.html

Defn: $x = y \mod z$ if $x - y$ is a multiple of z

Examples mod 12:

Defn: $x = y \mod z$ if $x - y$ is a multiple of z

Examples mod 12:

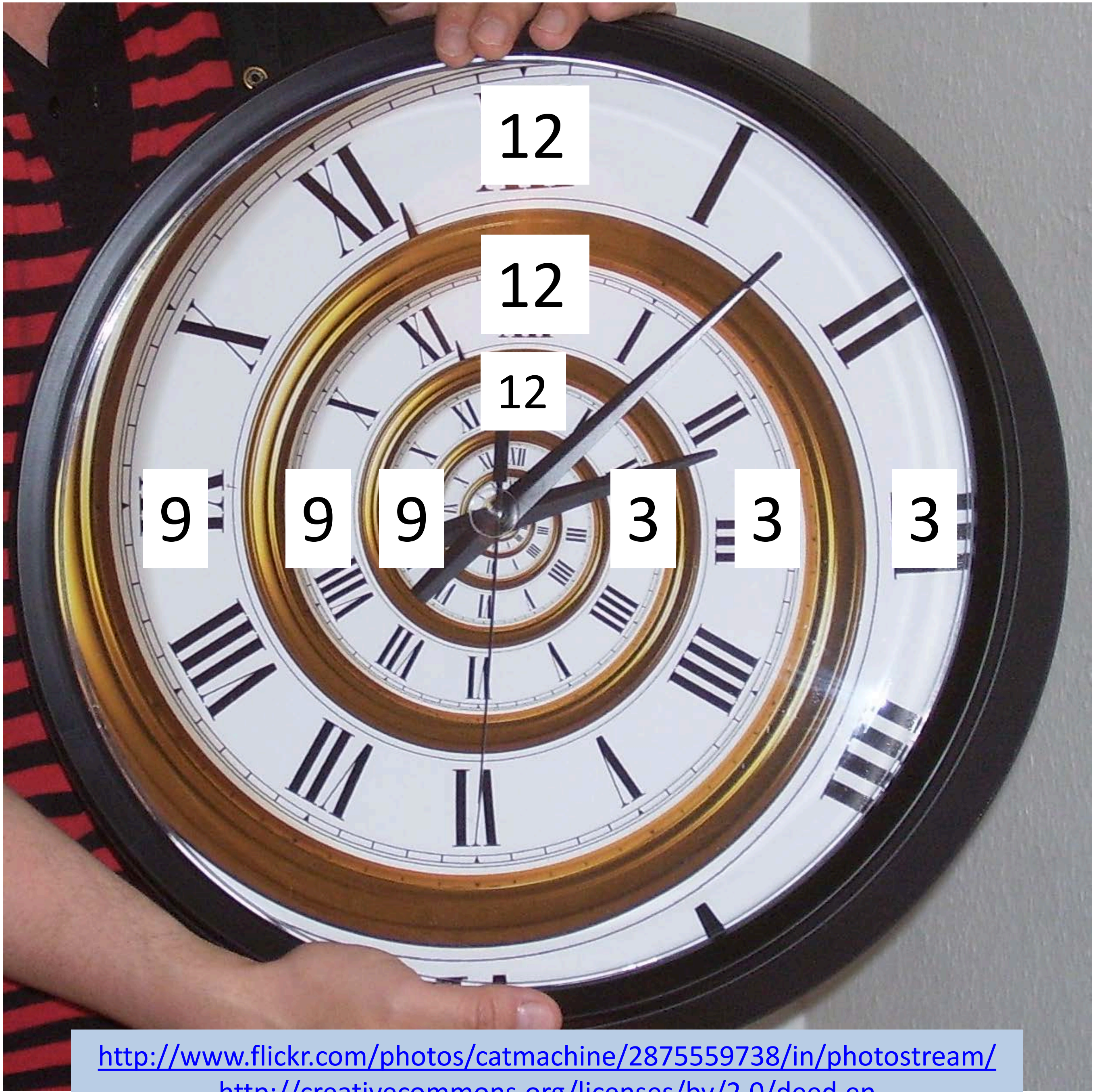
$3 = 15 \mod 12$ since $15 - 3 = 12$ is a multiple of 12

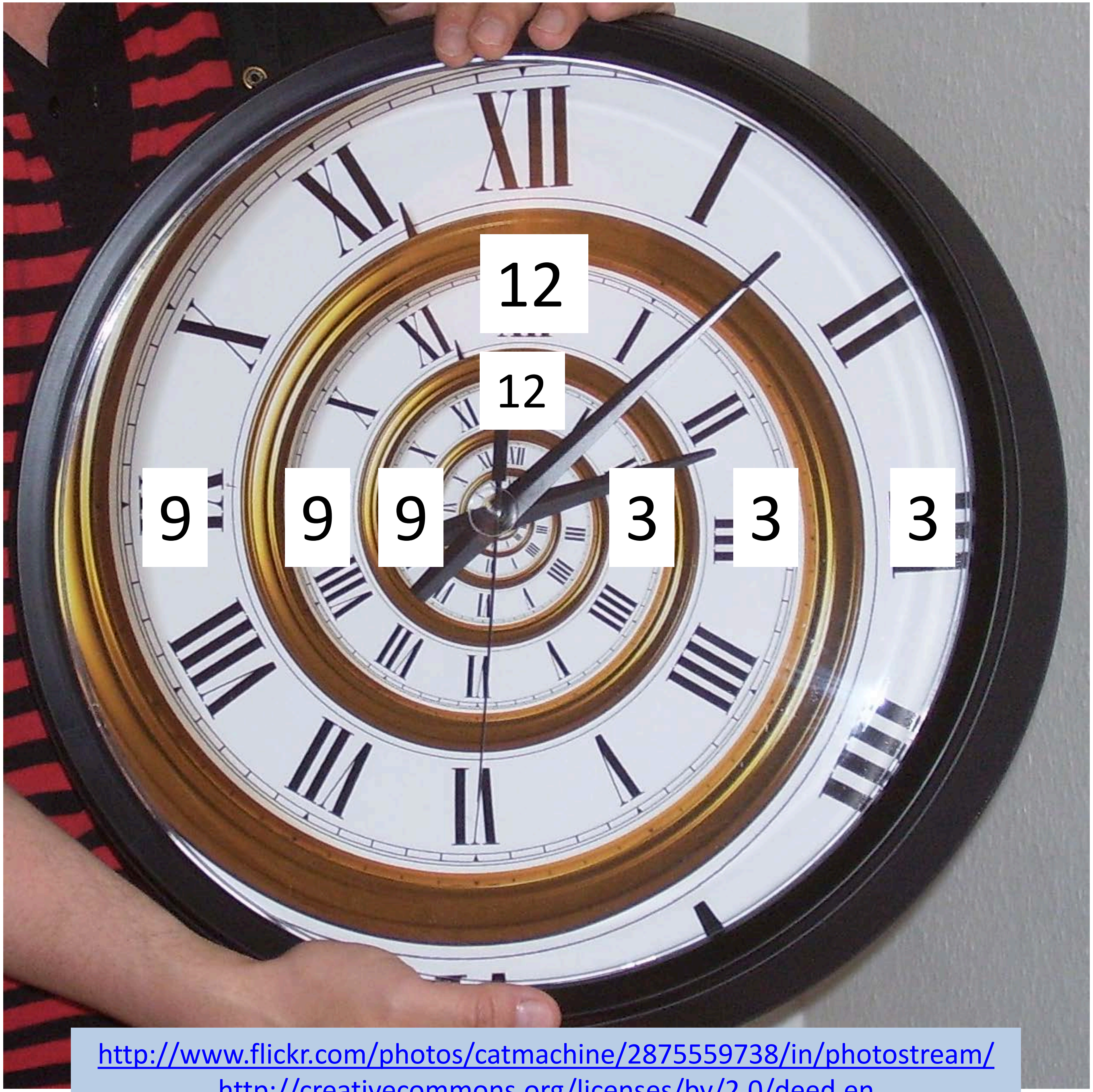
$3 = 27 \mod 12$ since $27 - 3 = 24$ is a multiple of 12

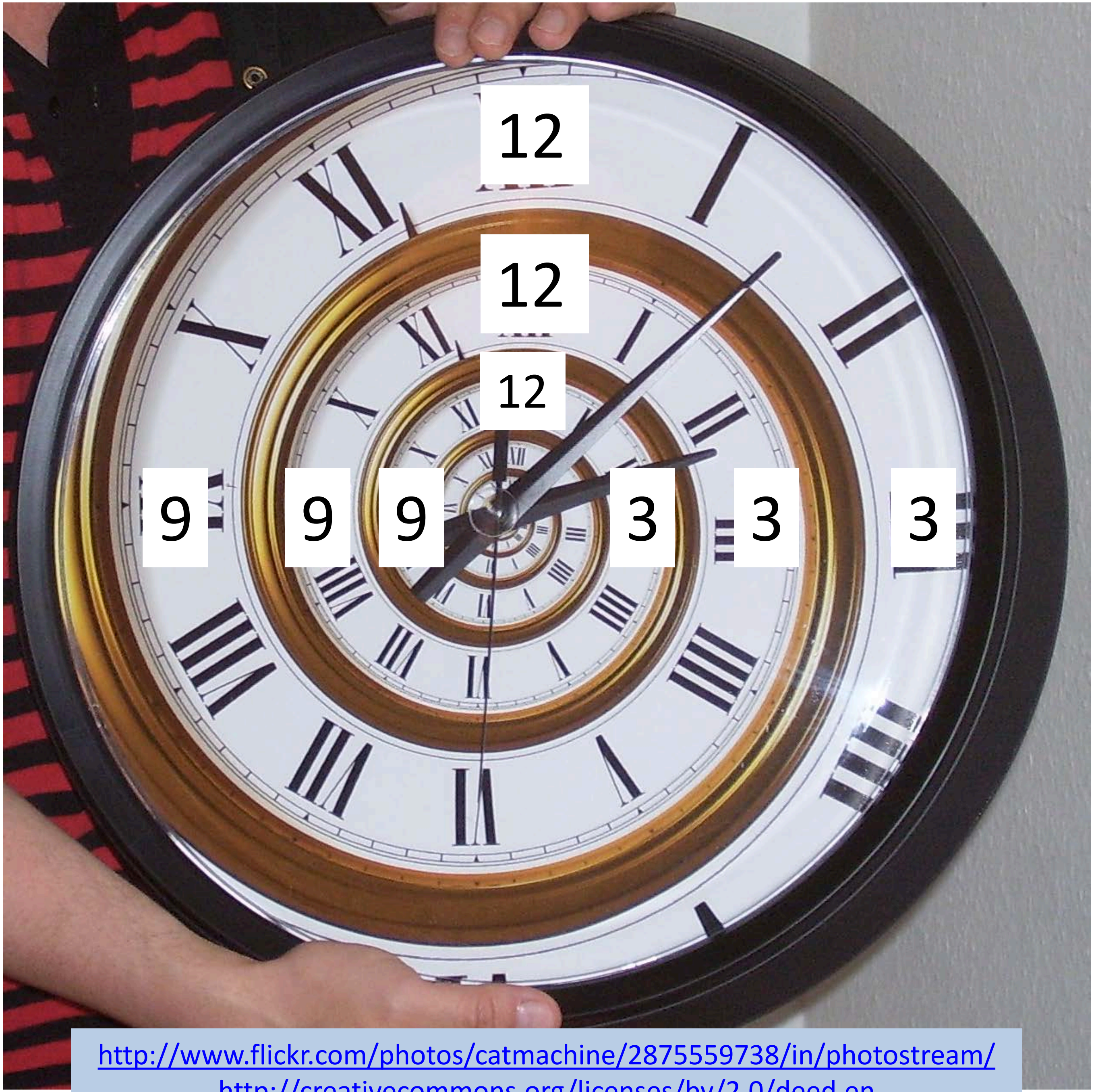
$3 = -9 \mod 12$ since $-9 - 3 = -12$ is a multiple of 12

$12 = 0 \mod 12$ since $12 - 0 = 12$ is a multiple of 12

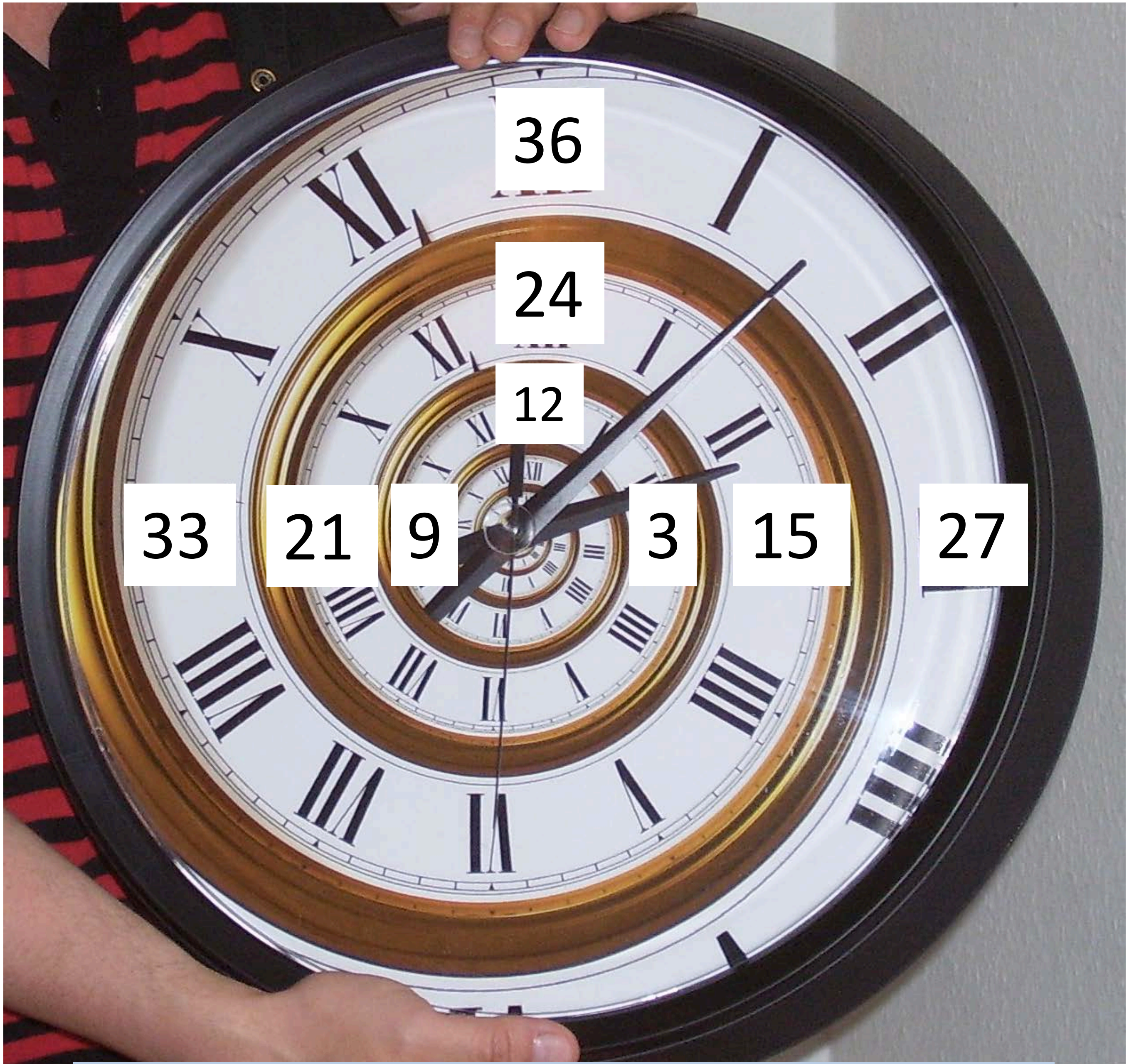
http://www.flickr.com/photos/catmachine/2875559738/in/photostream/


12
12
12
9
9
9
3
3
3


12
12
9
9
9
3
3
3



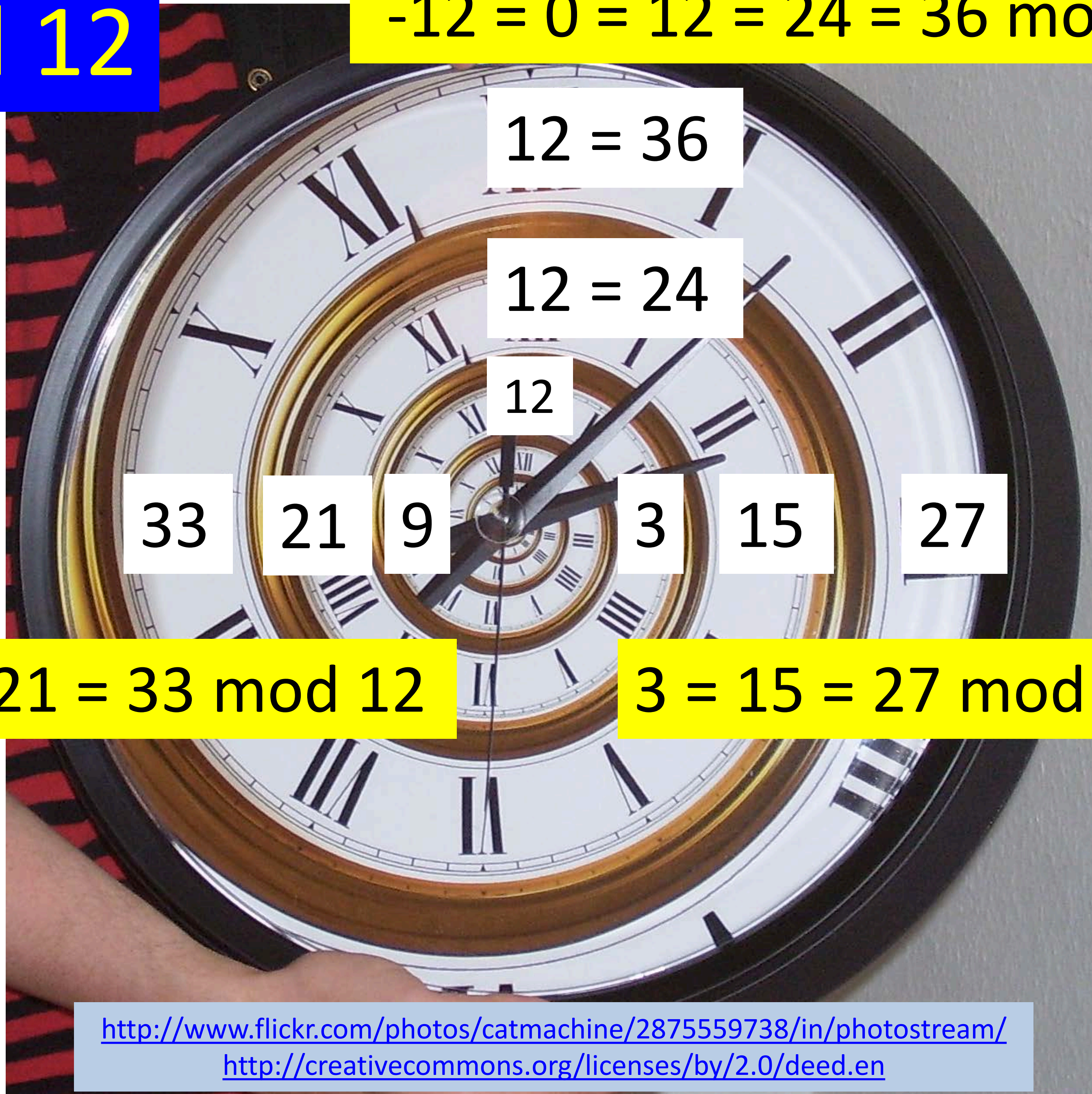
http://www.flickr.com/photos/catmachine/2875559738/in/photostream/



http://www.flickr.com/photos/catmachine/2875559738/in/photostream/

mod 12

-12 = 0 = 12 = 24 = 36 mod 12



9 = 21 = 33 mod 12

3 = 15 = 27 mod 12

mod 12

twelve = XII = 12 = 24 = 36 = 48 = -12 = -24 = 0

mod 12: $\mathbf{Z}_{12} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

Addition: $13 + 12 = 1 + 0 = 1 \bmod 12$

$15 + 72 = 3 + 0 = 3 \bmod 12$

$23 - 16 = 11 - 4 = -1 + 8 = -1 - 4 = -5 = 7 \bmod 12$

Sidenote: $3 \times 4 = 0 \bmod 12$

Video insert: mod 2 = light switch

Mod 2 arithmetic can be illustrated via a light switch

0 = no light

1 = light

1 + 1 = 0 = no light

1 + 1 + 1 = 1 = light

1 + 1 + 1 + 1 = 0 = no light

1 = 3 = 5 = 7 = light

0 = 2 = 4 = 6 = no light

mod 2

two = II = 2 = 4 = 6 = 8 = -2 = -4 = 0 mod 2

i.e., any even number mod 2 = zero

one = I = 1 = 3 = 5 = 7 = -1 = -3 = -5 = -7

i.e., any odd number mod 2 = one

mod 2: $\mathbf{Z_2} = \mathbf{Z/2Z} = \{0, 1\}$

Defn: $x = y \mod z$ if $x - y$ is a multiple of z

Examples:

$0 = 2 \mod 2$ since $2 - 0 = 2$ is a multiple of 2

$0 = -8 \mod 2$ since $-8 - 0 = -8$ is a multiple of 2

$1 = 3 \mod 2$ since $3 - 1 = 2$ is a multiple of 2

$1 = -1 \mod 2$ since $1 - (-1) = 2$ is a multiple of 2

Addition modulo 2

two = II = 2 = 4 = 6 = 8 = -2 = -4 = 0 mod 2

i.e., any even number mod 2 = zero

one = 1 = 3 = 5 = 7 = -1 = -3 = -5 = -7

i.e., any odd number mod 2 = one

mod 2: $\mathbf{Z_2} = \{0, 1\}$

Addition: even + even = even = 0

$4 - 10 = 0 + 0 = 0$

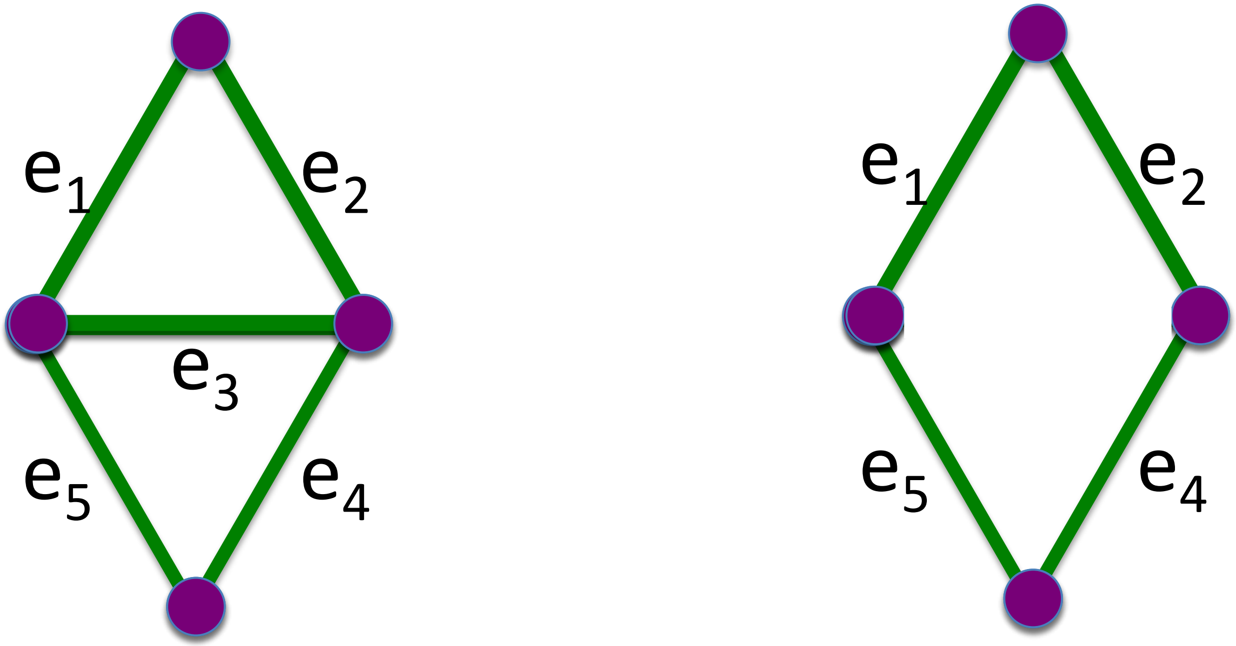
even + odd = odd

$24 + 15 = 1 + 0 = 1$

odd + odd = even = 0

$1 + 1 = 0$

With $\mathbf{Z_2} = \{0, 1\}$ coefficients:



$$(e_1 + e_2 + e_3) + (e_3 + e_4 + e_5) = e_1 + e_2 + 2e_3 + e_4 + e_5$$

$$= e_1 + e_2 + e_4 + e_5 \mod 2$$