MATH:7450 (22M:305) Topics in Topology: Scientific and Engineering Applications of Algebraic Topology

Sept 6, 2013: Calculating homology using matrices

Fall 2013 course offered through the University of Iowa Division of Continuing Education

Isabel K. Darcy, Department of Mathematics
Applied Mathematical and Computational Sciences,
University of Iowa

http://www.math.uiowa.edu/~idarcy/AppliedTopology.html http://www.ima.umn.edu/2008-2009/ND6.15-26.09

New Directions Short Course
Applied Algebraic Topology

June 15-26, 2009

Organizers

Gunnar Carlsson

Robert Ghrist

"Homology 2" morse, morse-conley, hodge & more: simple applications

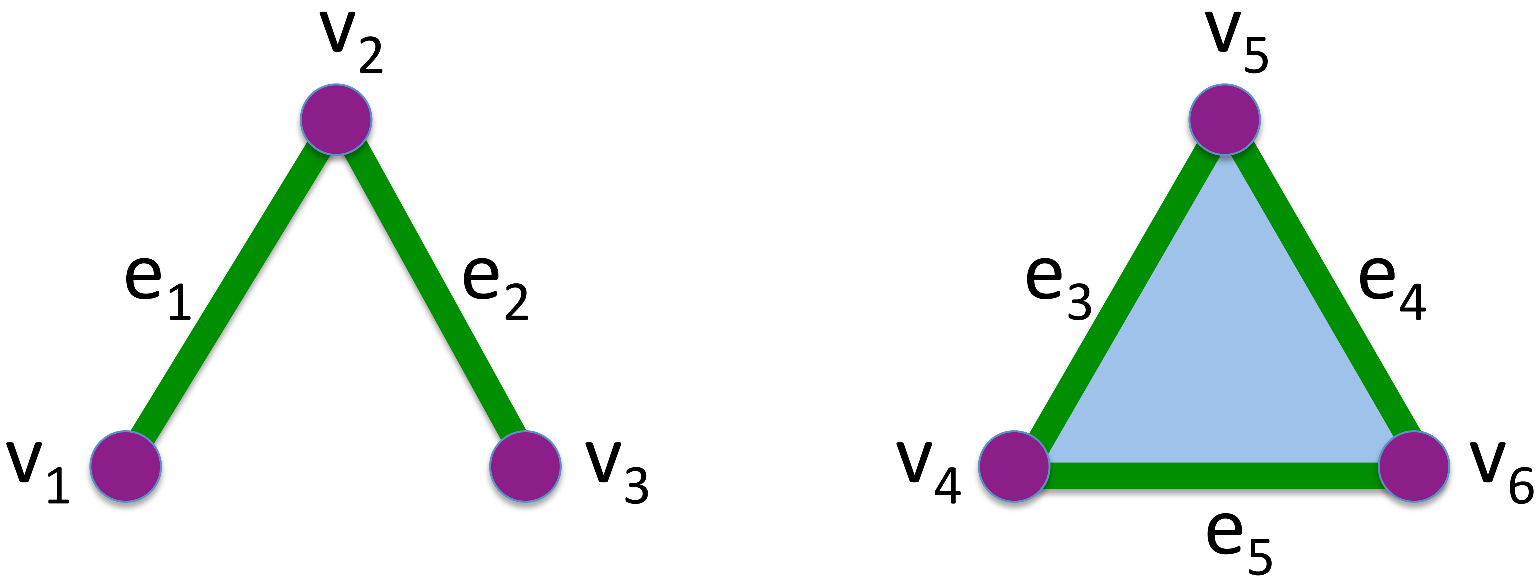
June 16, 2009 11:00 am - 12:30 pm

Robert Ghrist - University of Pennsylvania

- Lecture 4 slides (pdf)
- titlepage.pdf (pdf)
- Video (flv)

http://www.ima.umn.edu/2008-2009/ND6.15-26.09/abstracts.html#8322

Counting number of connected components using homology



$H_0 = Z_0/B_0 = \langle v_1, v_2, v_3, v_4, v_5, v_6 : v_1 + v_2 = 0, v_2 + v_3 = 0,$
$v_4 + v_5 = 0, v_5 + v_6 = 0, v_4 + v_6 = 0\rangle$

$H_0 = Z_0/B_0 = \langle [v_1], [v_4]\rangle$ where $[v_1] = \{v_1, v_2, v_3\}$

and $[v_4] = \{v_4, v_5, v_6\}$

Counting number of connected components using homology

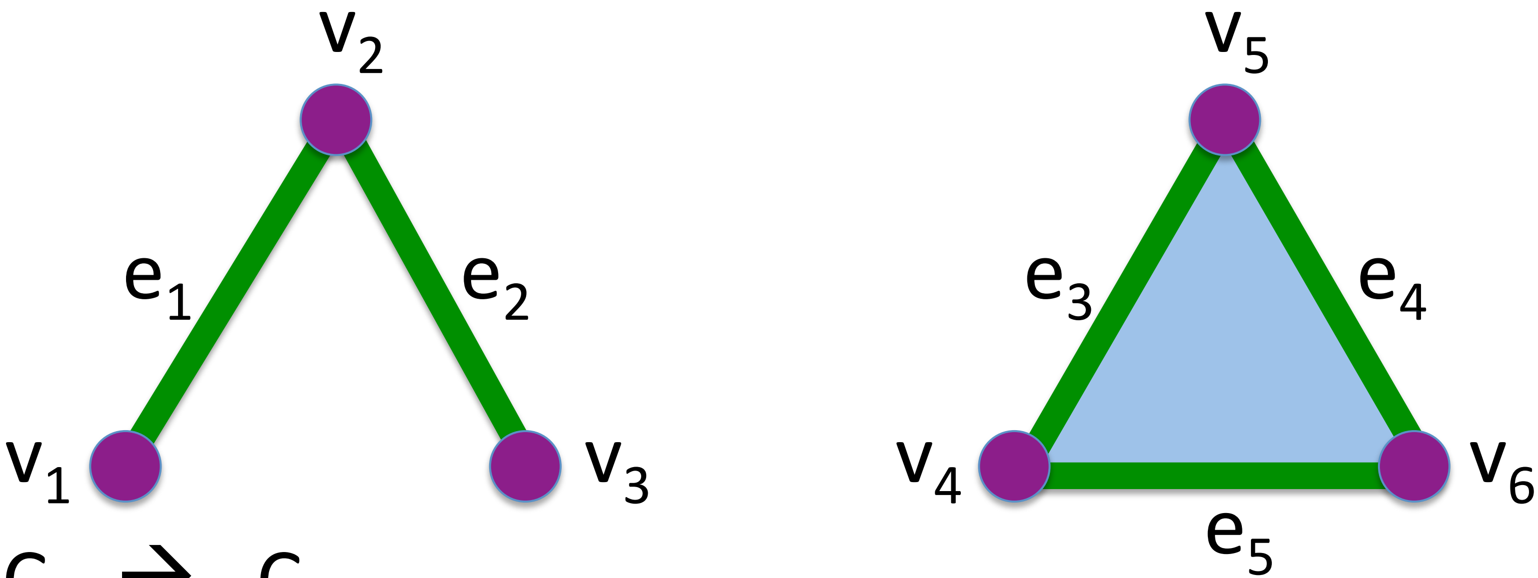
$Z_0/B_0 = \langle v_1, v_2, v_3, v_4, v_5, v_6 : v_1 + v_2 = 0, v_2 + v_3 = 0, v_4 + v_5 = 0, v_5 + v_6 = 0, v_4 + v_6 = 0 \rangle$

Use matrices:

$$
\begin{array}{c} \\ \{v_1, v_2\} \\ \{v_2, v_3\} \\ \{v_4, v_5\} \\ \{v_5, v_6\} \\ \{v_4, v_6\} \end{array}
\begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\
\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \end{array}
$$

See Computing Persistent Homology by Afra Zomorodian, Gunnar Carlsson

Counting number of connected components using homology



$\partial : C_1 \rightarrow C_0$

$\partial(e_1) = v_1 + v_2$

$\partial(e_2) = v_2 + v_3$

$\partial(e_3) = v_4 + v_5$

$\partial(e_4) = v_5 + v_6$

$\partial(e_5) = v_4 + v_6$

Extend linearly:

$\partial(\Sigma n_i e_i) = n_i \Sigma \partial(e_i)$

Counting number of connected components using homology

$Z_0/B_0 = \langle v_1, v_2, v_3, v_4, v_5, v_6 : v_1 + v_2 = 0, v_2 + v_3 = 0, v_4 + v_5 = 0, v_5 + v_6 = 0, v_4 + v_6 = 0 \rangle$

Use matrices:

$$
\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array}
\begin{array}{c}
\begin{array}{ccccc} \{v_1, v_2\} & \{v_2, v_3\} & \{v_4, v_5\} & \{v_5, v_6\} & \{v_4, v_6\} \end{array} \\
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1
\end{pmatrix}
\end{array}
$$

See Computing Persistent Homology by Afra Zomorodian, Gunnar Carlsson

Let $e_1 = \{v_1, v_2\} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Let $e_2 = \{v_2, v_3\} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Let $e_3 = \{v_4, v_5\} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

Let $e_4 = \{v_5, v_6\} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

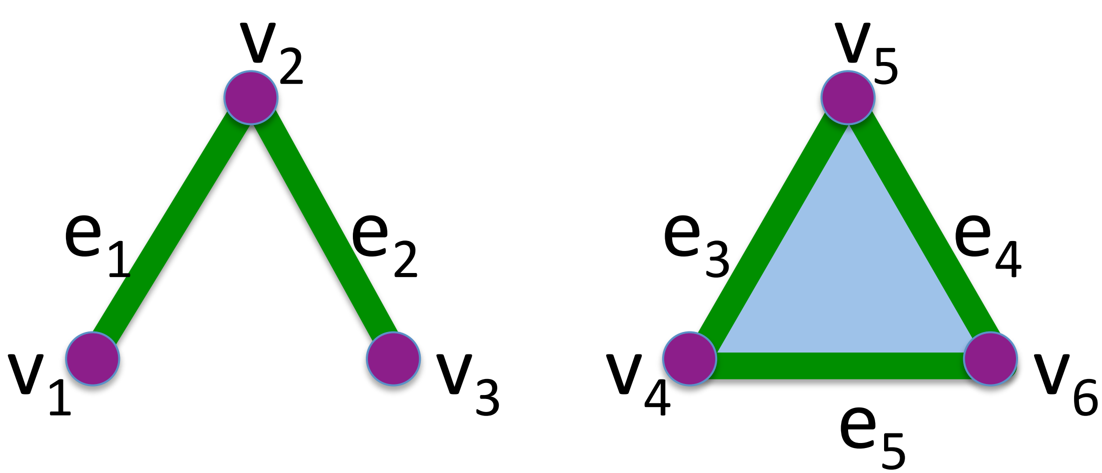
Let $e_5 = \{v_4, v_6\} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, v_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, v_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$v_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

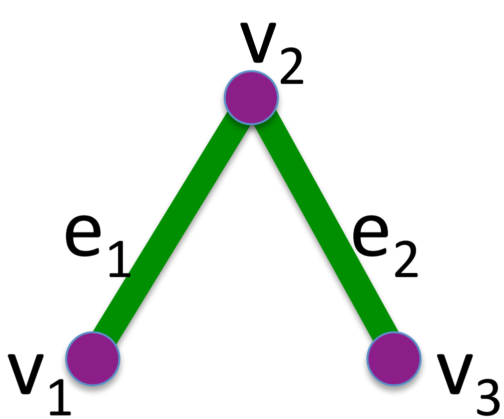
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

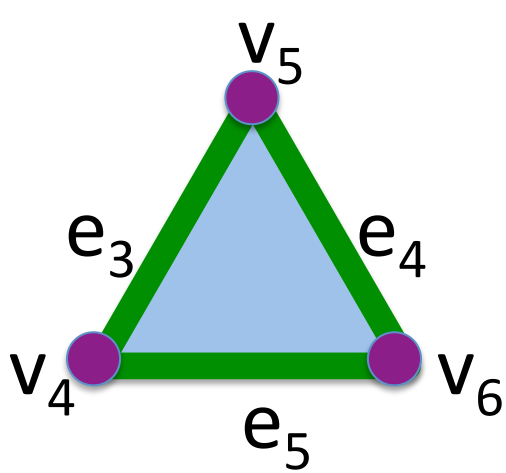
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



$$\partial(e_1) = v_1 + v_2$$

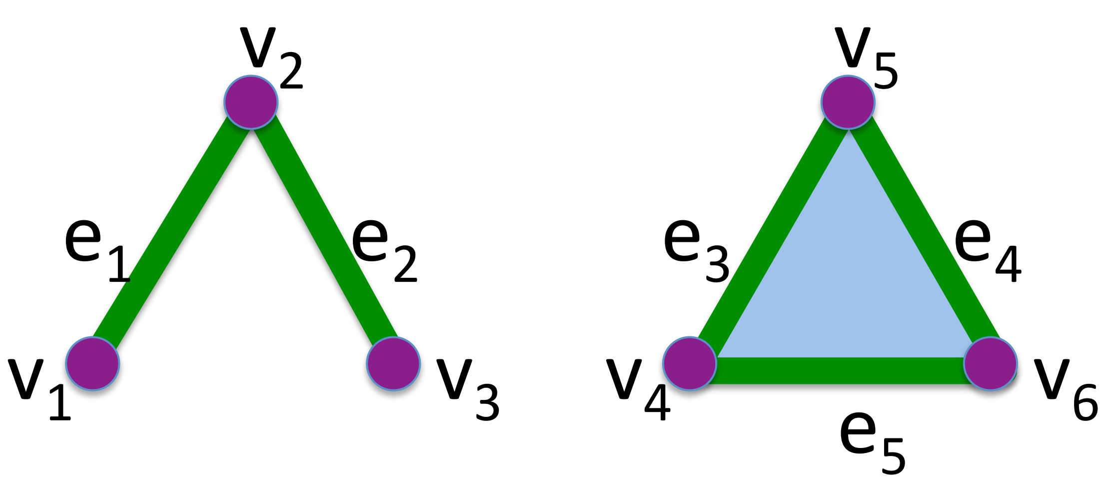
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$





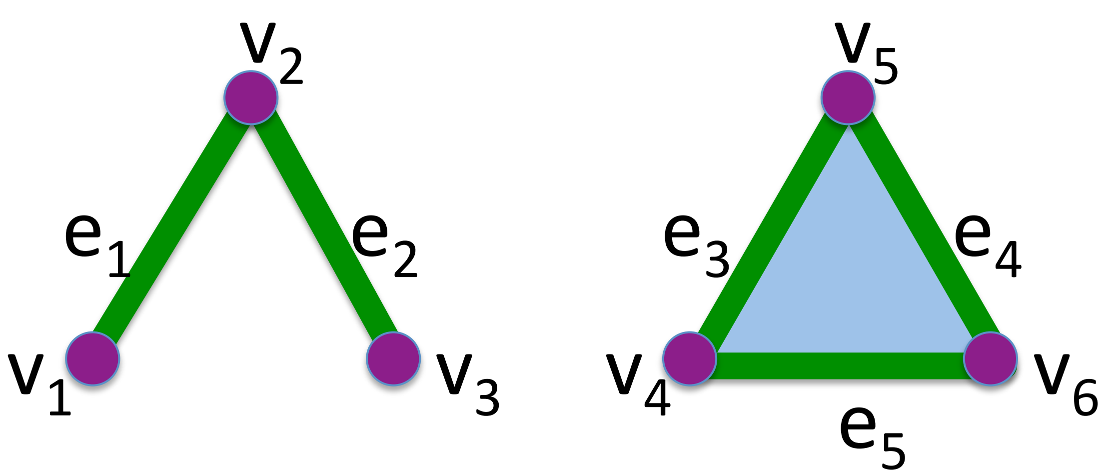
$$\partial(e_2) = v_2 + v_3$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$



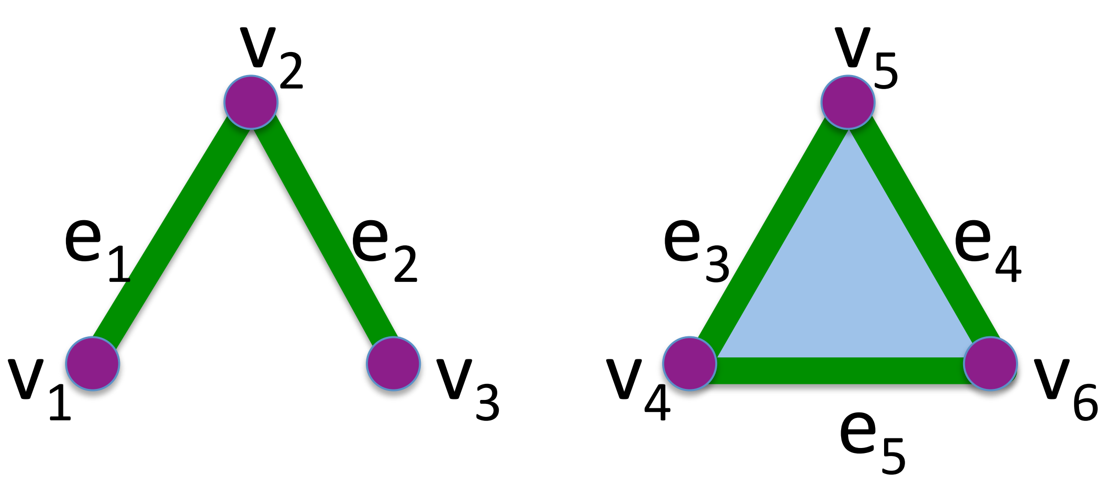
$$\partial(e_3) = v_4 + v_5$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$



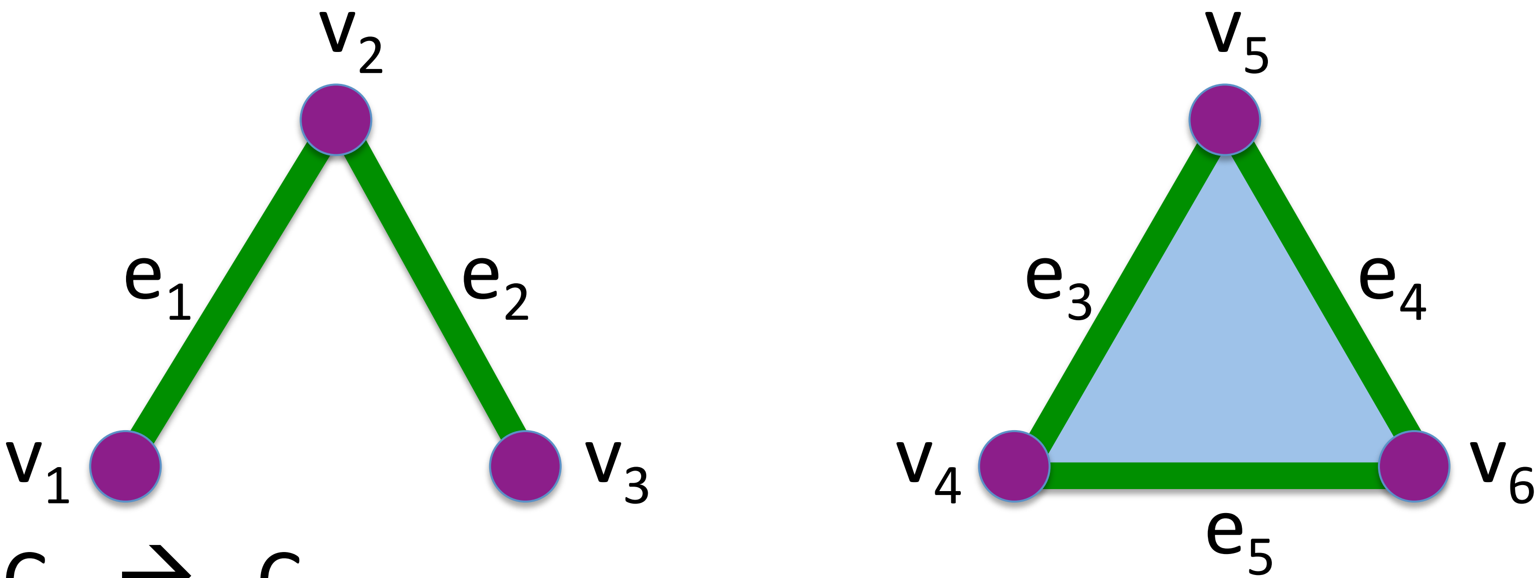
$$\partial(e_4) = v_5 + v_6$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$



$$\partial(e_5) = v_4 + v_6$$

Counting number of connected components using homology



$\partial : C_1 \rightarrow C_0$

$\partial(e_1) = v_1 + v_2$

$\partial(e_2) = v_2 + v_3$

$\partial(e_3) = v_4 + v_5$

$\partial(e_4) = v_5 + v_6$

$\partial(e_5) = v_4 + v_6$

Extend linearly:

$\partial(\Sigma n_i e_i) = n_i \Sigma \partial(e_i)$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \end{bmatrix}$$

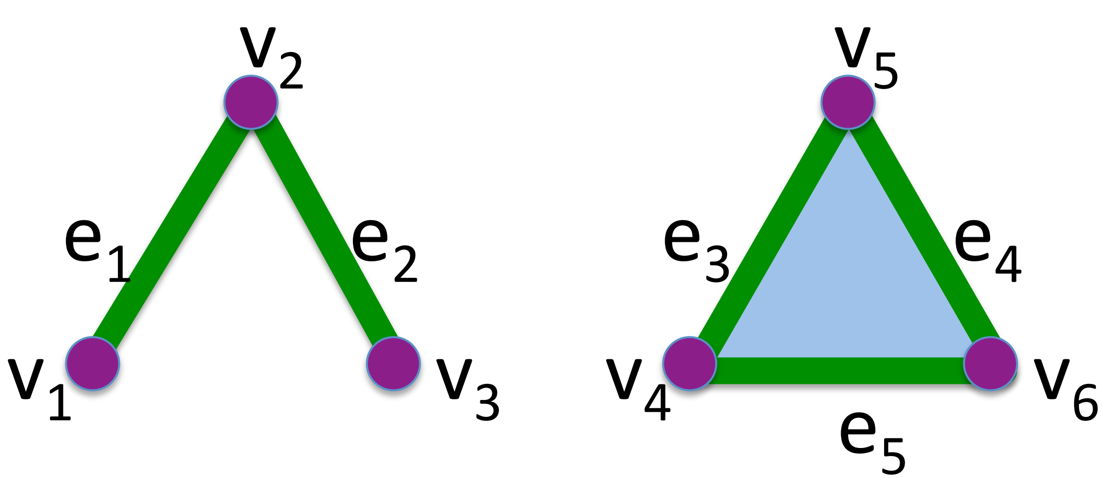
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \end{bmatrix} =$$

$$n_1 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + n_2 \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + n_3 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} + n_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} + n_5 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

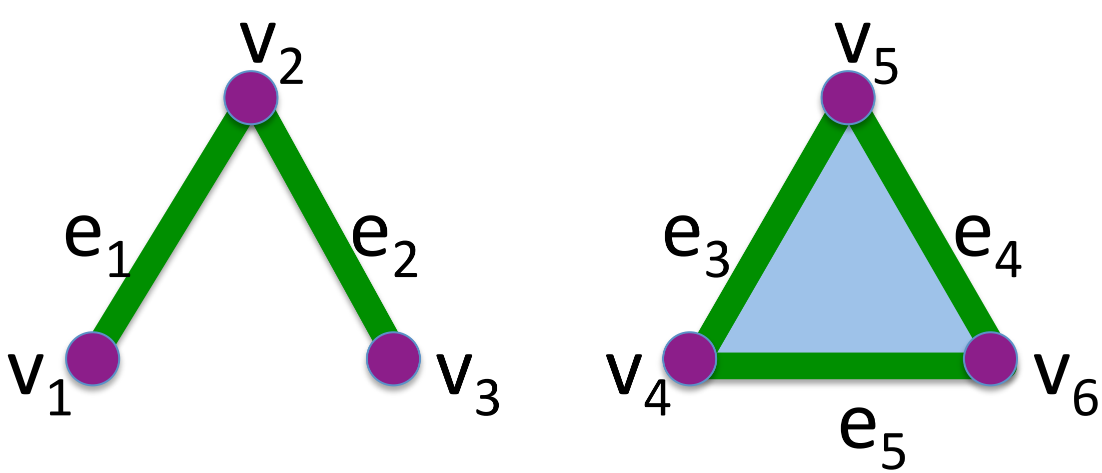
$$n_1 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + n_2 \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + n_3 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} + n_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} + n_5 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

B_0 = Image of ∂_1 = column space of $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$

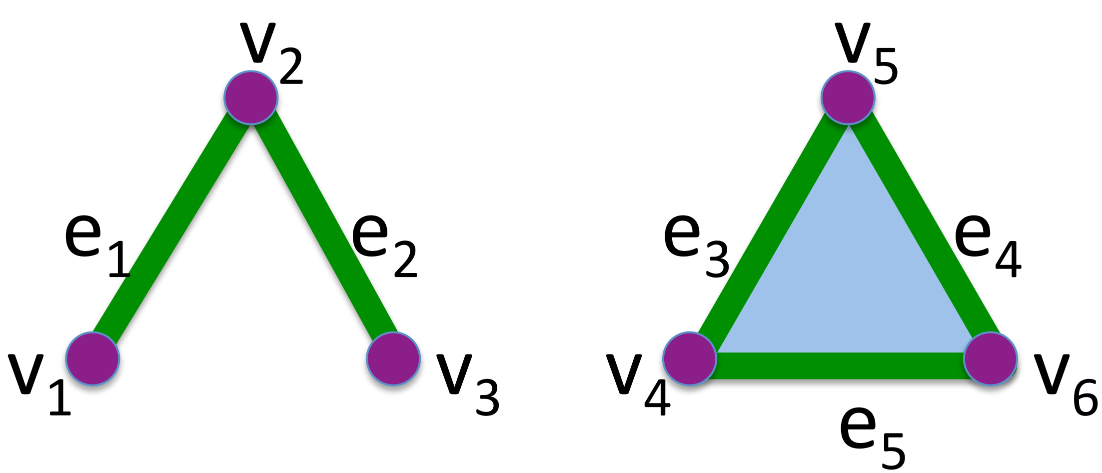
	$\{v_1, v_2\}$	$\{v_2, v_3\}$	$\{v_4, v_5\}$	$\{v_5, v_6\}$	$\{v_4, v_6\}$
v_1	1	0	0	0	0
v_2	1	1	0	0	0
v_3	0	1	0	0	0
v_4	0	0	1	0	1
v_5	0	0	1	1	0
v_6	0	0	0	1	1



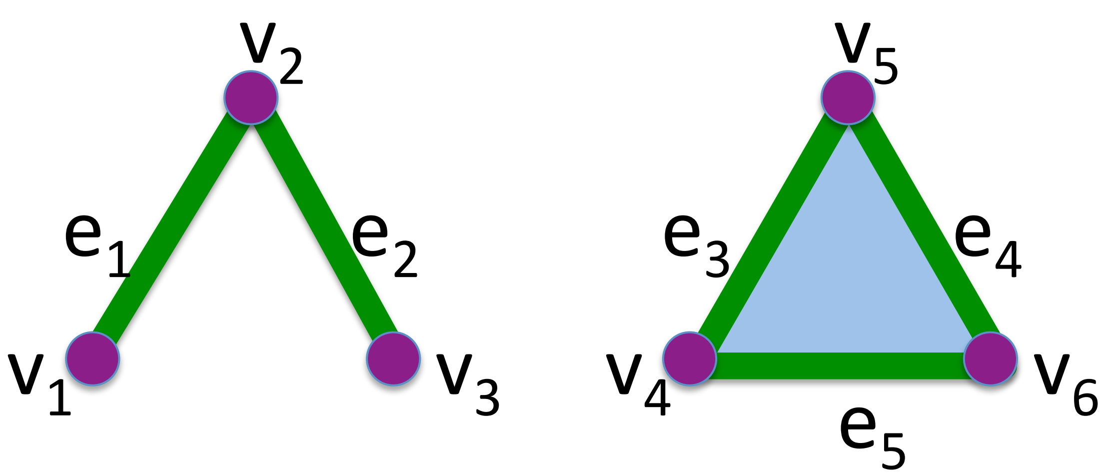
$$
\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array}
\begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\
\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \end{array}
$$



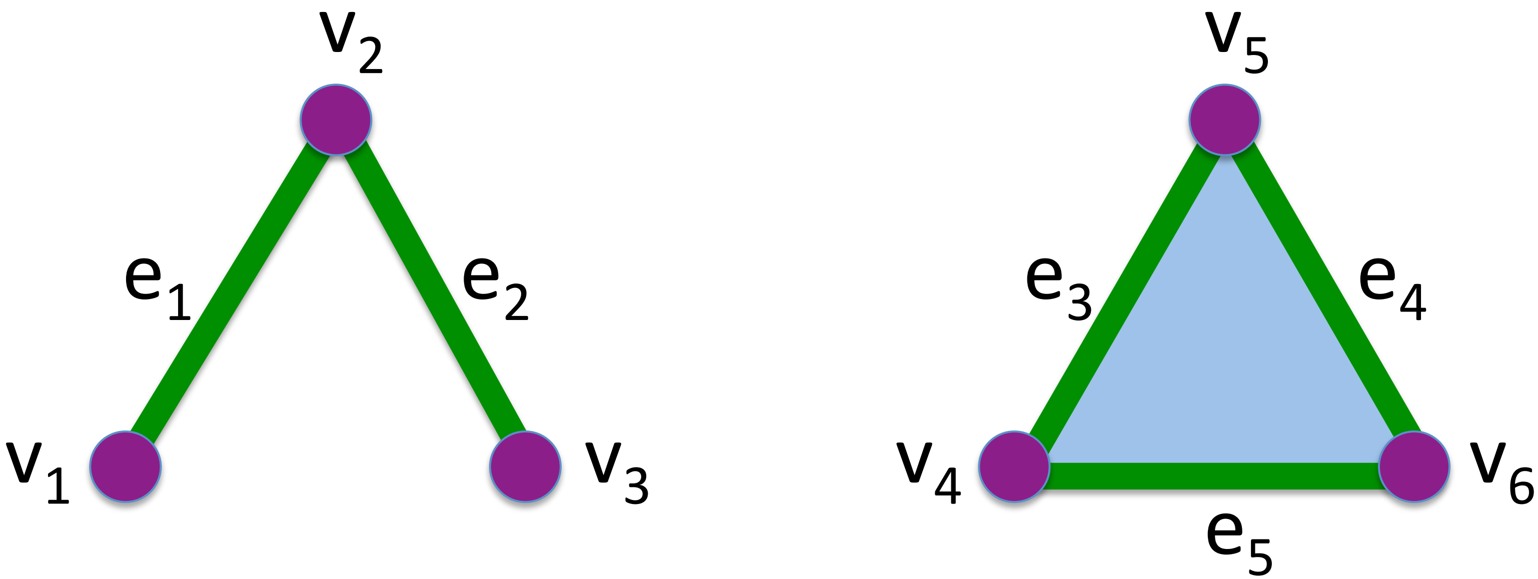
$$
\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array}
\begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_3 + e_5 \end{array} \\
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1
\end{pmatrix} \end{array}
$$



$$
\begin{array}{c@{\;}c}
 & \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_3 + e_4 + e_5 \end{array} \\
\begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} &
\left(\begin{array}{ccccc}
1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 0
\end{array} \right)
\end{array}
$$



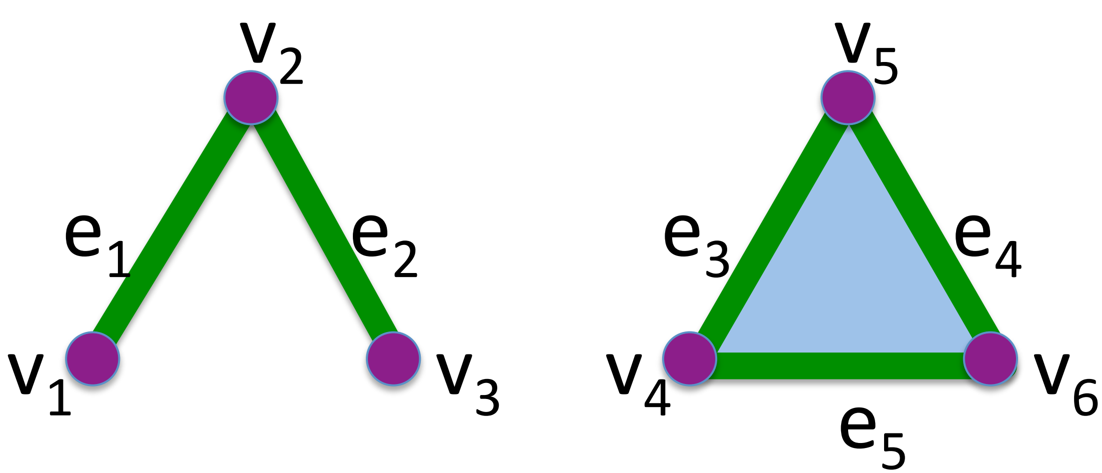
Counting number of connected components using homology



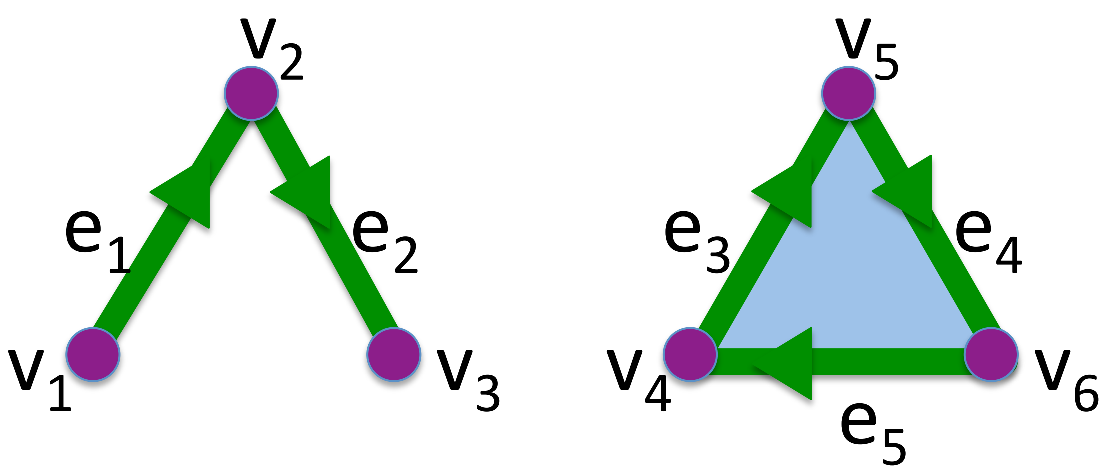
$H_0 = Z_0/B_0 = \langle v_1, v_2, v_3, v_4, v_5, v_6 : v_1 + v_2 = 0, v_2 + v_3 = 0, v_4 + v_5 = 0, v_5 + v_6 = 0, v_4 + v_6 = 0 \rangle$

$H_0 = Z_0/B_0 = \langle [v_1], [v_4] \rangle$ where $[v_1] = \{v_1, v_2, v_3\}$ and $[v_4] = \{v_4, v_5, v_6\}$

$$
\begin{array}{c c}
 & \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_3+e_4+e_5 \end{array} \\
\begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} &
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 0
\end{pmatrix}
\end{array}
$$

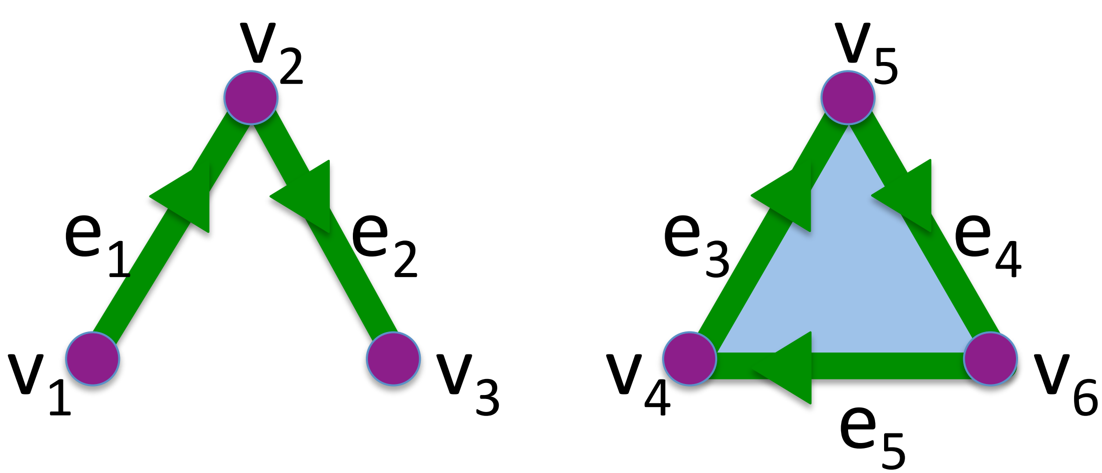


$$
\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array}
\begin{array}{c}
\begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\
\begin{pmatrix}
-1 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 1 \\
0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 1 & -1
\end{pmatrix}
\end{array}
$$



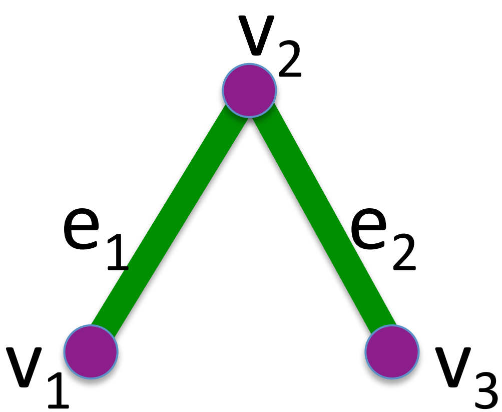
Using arbitrary coefficients

$$
\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array}
\begin{array}{c}
\begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_3 + e_4 + e_5 \end{array} \\
\begin{pmatrix}
-1 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 1 & 0
\end{pmatrix}
\end{array}
$$

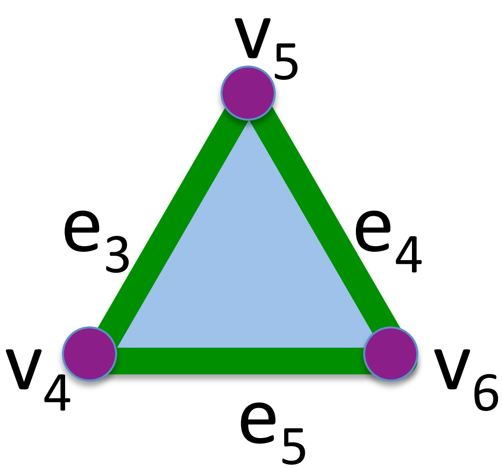


Using arbitrary coefficients

Row operations



$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_3+e_4+e_5 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$



$$\begin{array}{c} \\ v_1+v_2 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_3+e_4+e_5 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

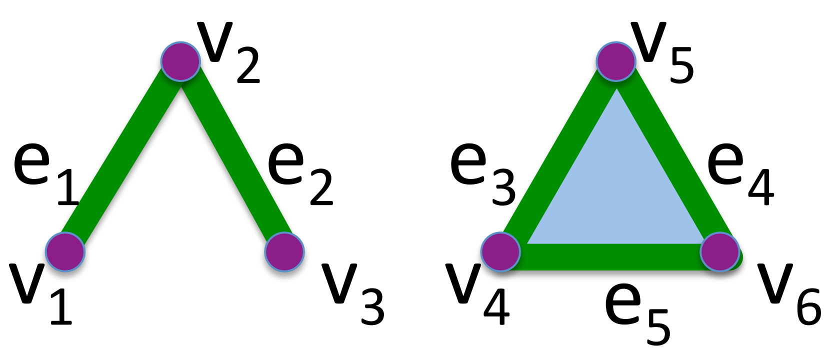
Row operations

	e_1	e_2	e_3	e_4	$e_3+e_4+e_5$
v_1	1	0	0	0	0
v_2	1	1	0	0	0
v_3	0	1	0	0	0
v_4	0	0	1	0	0
v_5	0	0	1	1	0
v_6	0	0	0	1	0

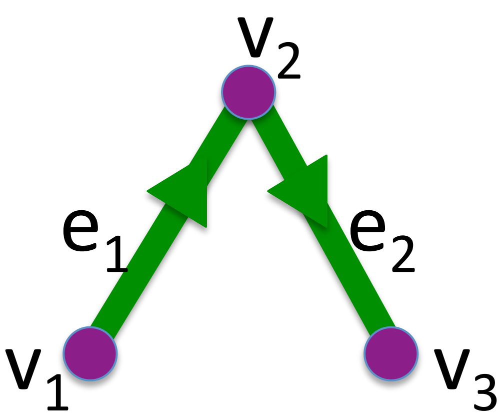
	e_1	e_2	e_3	e_4	$e_3+e_4+e_5$
v_1+v_2	1	0	0	0	0
v_2	0	1	0	0	0
v_3	0	1	0	0	0
v_4	0	0	1	0	0
v_5	0	0	1	1	0
v_6	0	0	0	1	0

	e_1	e_2	e_3	e_4	$e_3+e_4+e_5$
v_1+v_2	1	0	0	0	0
v_2+v_3	0	1	0	0	0
v_3	0	0	0	0	0
v_4+v_5	0	0	1	0	0
v_5	0	0	0	1	0
v_6	0	0	0	1	0

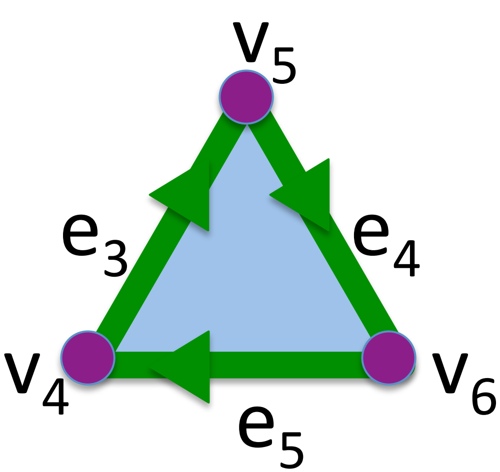
	e_1	e_2	e_3	e_4	$e_3+e_4+e_5$
v_1+v_2	1	0	0	0	0
v_2+v_3	0	1	0	0	0
v_3	0	0	0	0	0
v_4+v_5	0	0	1	0	0
v_5+v_6	0	0	0	1	0
v_6	0	0	0	0	0



Row operations using arbitrary coefficients

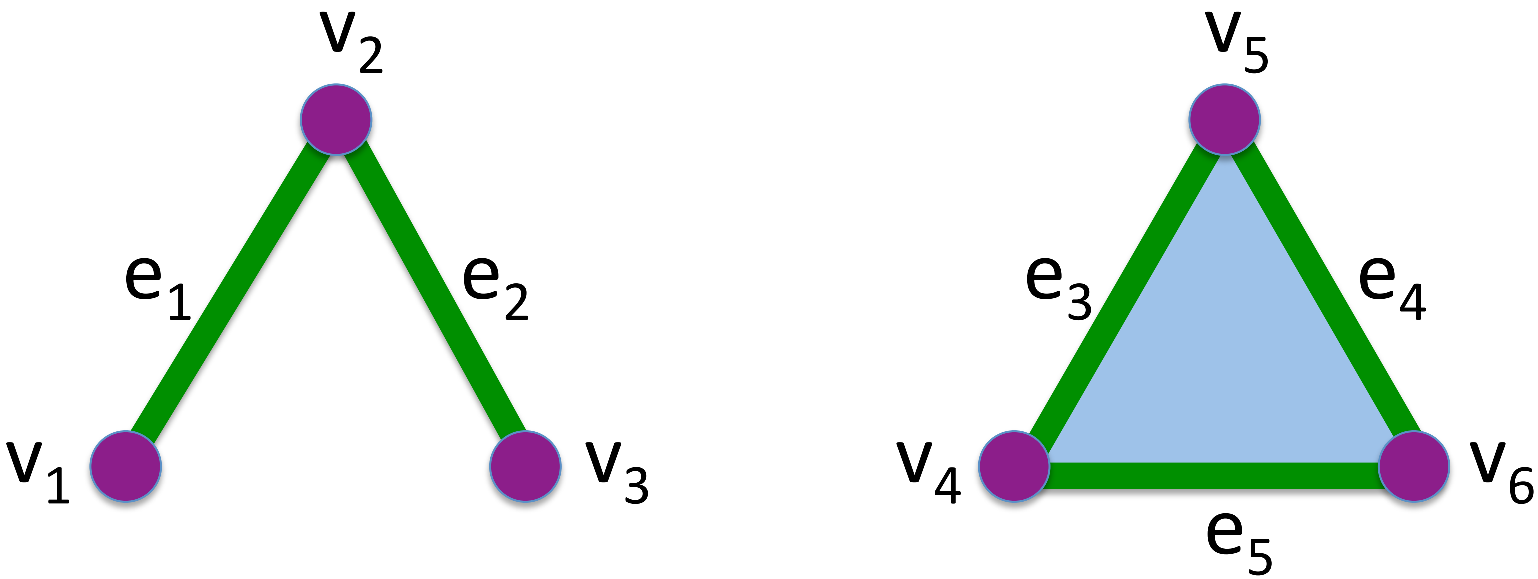


$$
\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array}
\begin{array}{c}
\begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_3+e_4+e_5 \end{array} \\
\begin{pmatrix}
-1 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 1 & 0
\end{pmatrix}
\end{array}
$$



$$
\begin{array}{c} \\ v_1 - v_2 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array}
\begin{array}{c}
\begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_3+e_4+e_5 \end{array} \\
\begin{pmatrix}
-1 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 1 & 0
\end{pmatrix}
\end{array}
$$

Counting number of connected components using homology

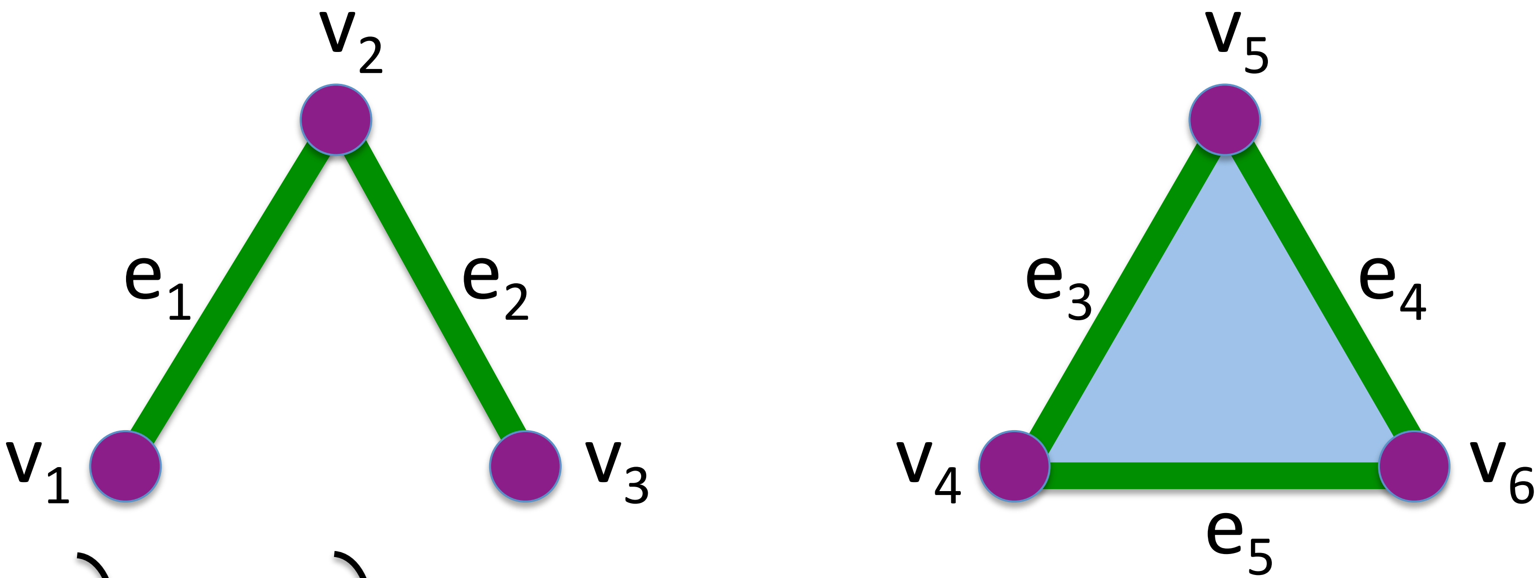


$H_0 = Z_0/B_0 = \langle v_1, v_2, v_3, v_4, v_5, v_6 : v_1 + v_2 = 0, v_2 + v_3 = 0,$
$v_4 + v_5 = 0, v_5 + v_6 = 0, v_4 + v_6 = 0\rangle$

$H_0 = Z_0/B_0 = \langle [v_1], [v_4]\rangle$ where $[v_1] = \{v_1, v_2, v_3\}$

and $[v_4] = \{v_4, v_5, v_6\}$

Counting number of connected components using homology



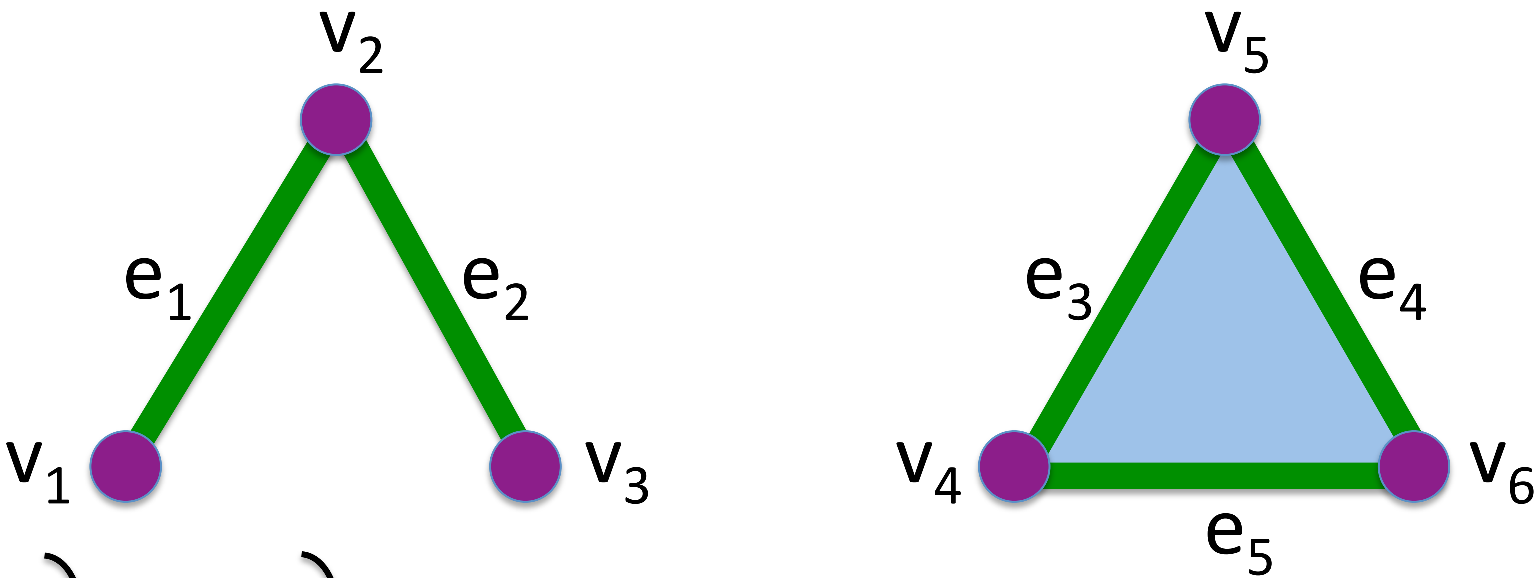
$$C_1 \xrightarrow{\partial_1} C_0 \xrightarrow{\partial_0} 0$$

Z_0 = kernel of ∂_0

$= \{x : \partial_0(x) = 0\}$

$= C_0 = \mathbf{Z_2}[v_1, v_2, v_3, v_4, v_5, v_6]$

Counting number of connected components using homology



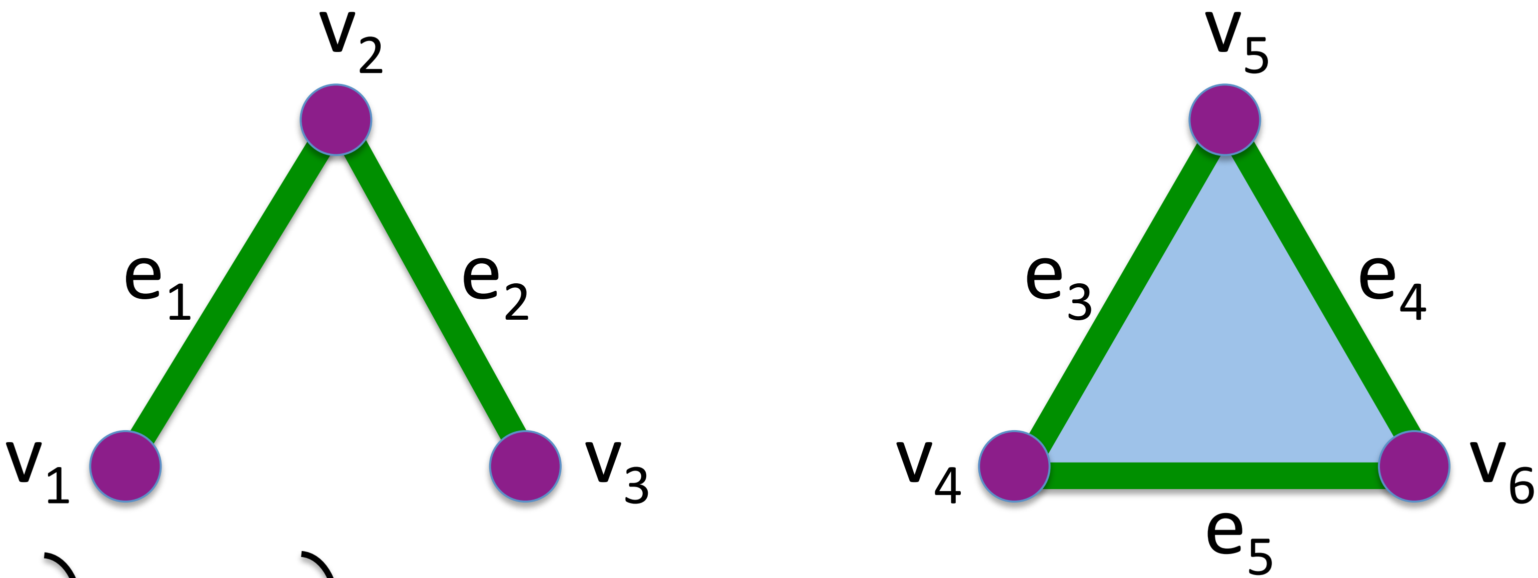
$$C_1 \xrightarrow{\partial_1} C_0 \xrightarrow{\partial_0} 0$$

Z_0 = kernel of ∂_0 = null space of M_0 =

= $\{x : \partial_0(x) = 0\}$

= $C_0 = \mathbf{Z_2}[v_1, v_2, v_3, v_4, v_5, v_6]$

Counting number of connected components using homology



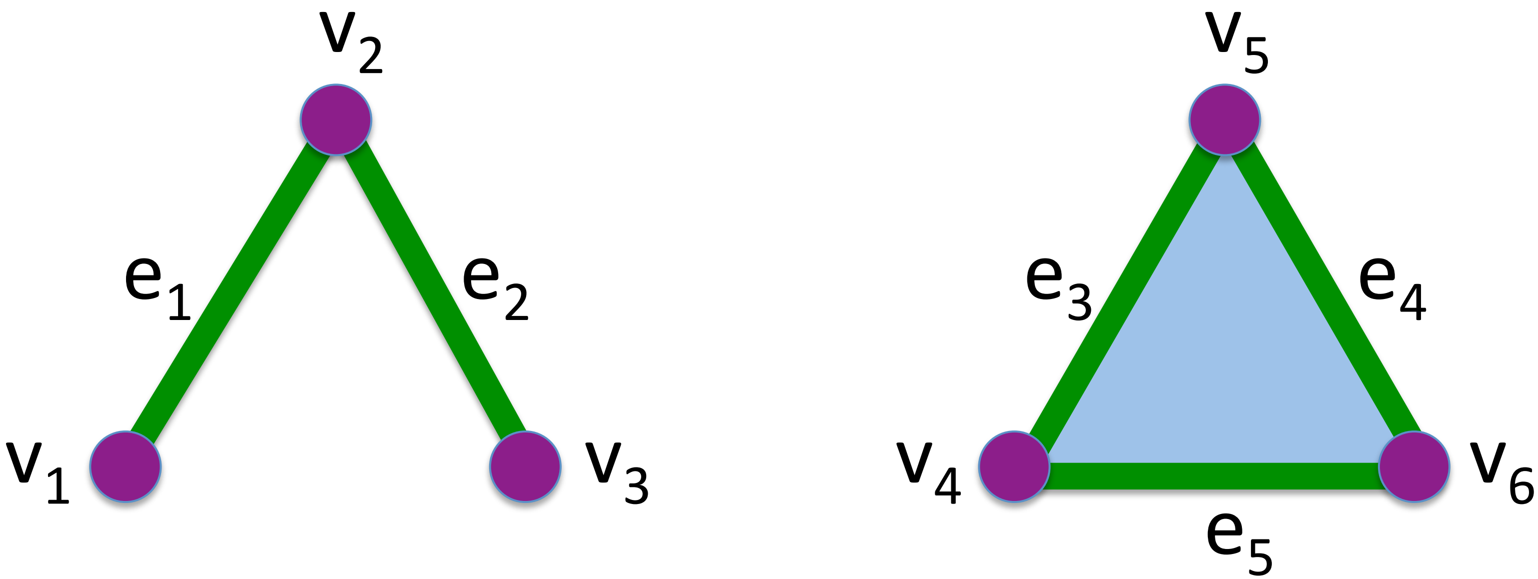
$$C_1 \xrightarrow{\partial_1} C_0 \xrightarrow{\partial_0} 0$$

Z_0 = kernel of ∂_0 = null space of $M_0 = [0\ 0\ 0\ 0\ 0\ 0]$

$= \{x : \partial_0(x) = 0\}$

$= C_0 = \mathbf{Z_2}[v_1, v_2, v_3, v_4, v_5, v_6]$

Counting number of connected components using homology



$H_0 = Z_0/B_0 = \langle v_1, v_2, v_3, v_4, v_5, v_6 : v_1 + v_2 = 0, v_2 + v_3 = 0,$
$v_4 + v_5 = 0, v_5 + v_6 = 0, v_4 + v_6 = 0\rangle$

$H_0 = Z_0/B_0 = \langle [v_1], [v_4]\rangle$ where $[v_1] = \{v_1, v_2, v_3\}$

and $[v_4] = \{v_4, v_5, v_6\}$

$$C_1 \xrightarrow{\partial_1} C_0 \xrightarrow{\partial_0} 0$$

$H_0 = Z_0/B_0$ = (kernel of ∂_0)/ (image of ∂_1)

$$= \frac{\text{null space of } M_0}{\text{column space of } M_1}$$

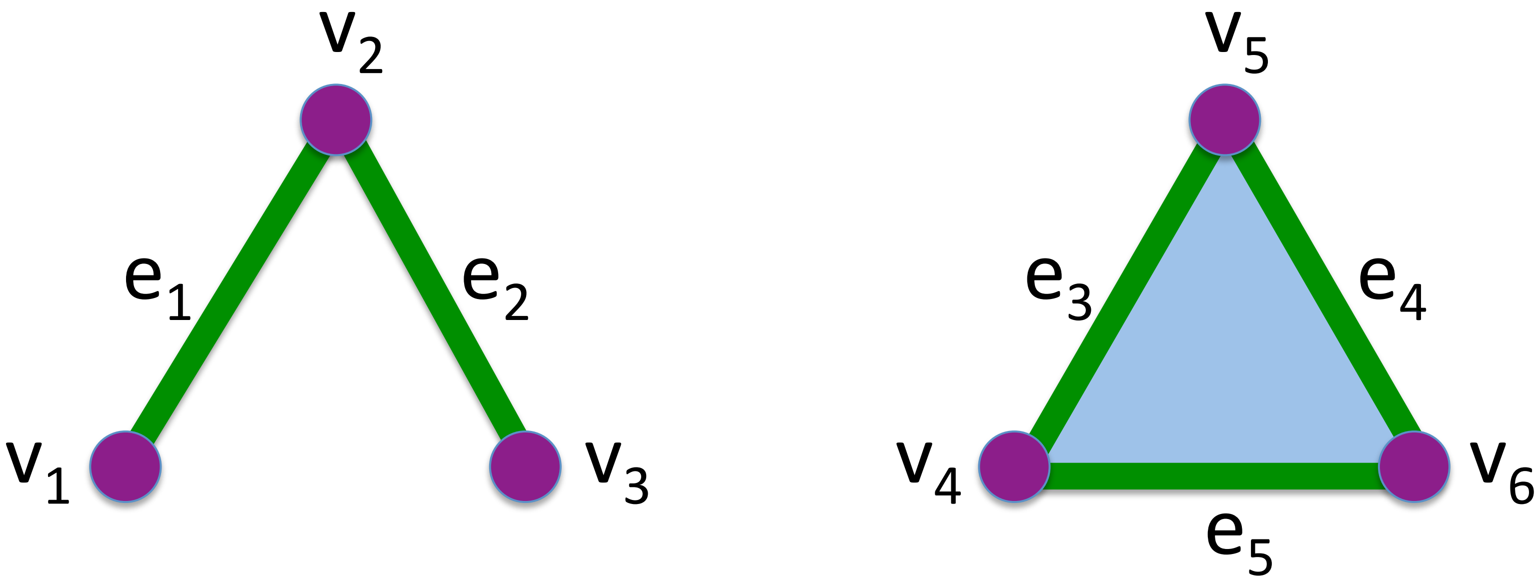
Rank H_0 = Rank Z_0 – Rank B_0

Z_0 = null space of [0 0 0 0 0 0]

B_0 = column space of

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_3+e_4+e_5 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

Counting number of connected components using homology



$H_0 = Z_0/B_0 = \langle v_1, v_2, v_3, v_4, v_5, v_6 : v_1 + v_2 = 0, v_2 + v_3 = 0,$
$v_4 + v_5 = 0, v_5 + v_6 = 0, v_4 + v_6 = 0\rangle$

$H_0 = Z_0/B_0 = \langle [v_1], [v_4] \rangle$ where $[v_1] = \{v_1, v_2, v_3\}$

and $[v_4] = \{v_4, v_5, v_6\}$

$$C_{n+1} \xrightarrow{\partial_{n+1}} C_n \xrightarrow{\partial_n} C_{n-1} \rightarrow \ldots \rightarrow C_2 \xrightarrow{\partial_2} C_1 \xrightarrow{\partial_1} C_0 \xrightarrow{\partial_0} 0$$

$$H_n = Z_n/B_n = (\text{kernel of } \partial_n)/(\text{image of } \partial_{n+1})$$

$$= \frac{\text{null space of } M_n}{\text{column space of } M_{n+1}}$$

$$\text{Rank } H_n = \text{Rank } Z_n - \text{Rank } B_n$$

$$C_2 \xrightarrow{\partial_2} C_1 \xrightarrow{\partial_1} C_0$$

$H_1 = Z_1/B_1 = (\text{kernel of } \partial_1)/(\text{image of } \partial_2)$

$$= \frac{\text{null space of } M_1}{\text{column space of } M_2}$$

$\text{Rank } H_1 = \text{Rank } Z_1 - \text{Rank } B_1$

$$C_2 \xrightarrow{\partial_2} C_1 \xrightarrow{\partial_1} C_0$$

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 \end{matrix} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \end{array}$$

Z_1 = kernel of ∂_1 = null space of M_1

$$C_2 \xrightarrow{\partial_2} C_1 \xrightarrow{\partial_1} C_0$$

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_3 + e_4 + e_5 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

Z_1 = kernel of ∂_1 = null space of M_1

$$C_2 \xrightarrow{\partial_2} C_1 \xrightarrow{\partial_1} C_0$$

$$\begin{array}{c c} & \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_3+e_4+e_5 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} & \left(\begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right) \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \end{bmatrix} \end{array}$$

Z_1 = kernel of ∂_1 = null space of M_1

$$C_2 \xrightarrow{\partial_2} C_1 \xrightarrow{\partial_1} C_0$$

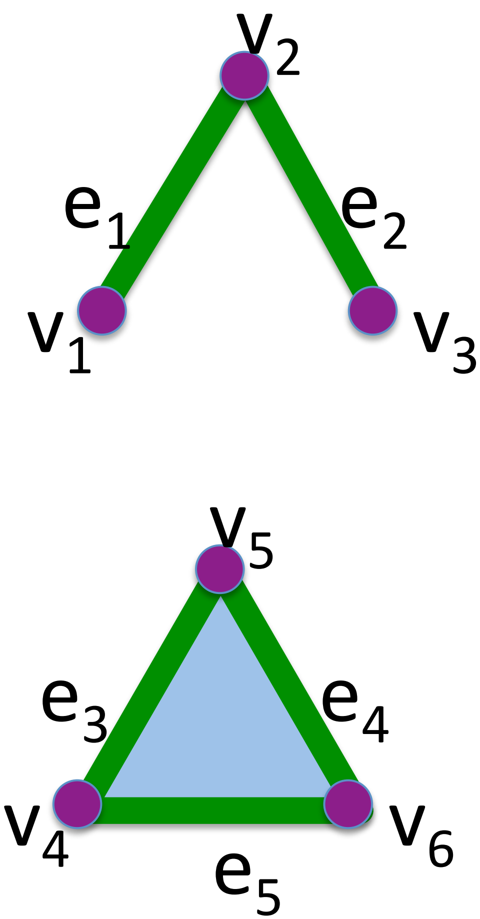
$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_3 + e_4 + e_5 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \end{bmatrix}$$

Z_1 = kernel of ∂_1 = null space of M_1

= $\langle e_3 + e_4 + e_5 \rangle$

$$C_2 \xrightarrow{\partial_2} C_1 \xrightarrow{\partial_1} C_0$$

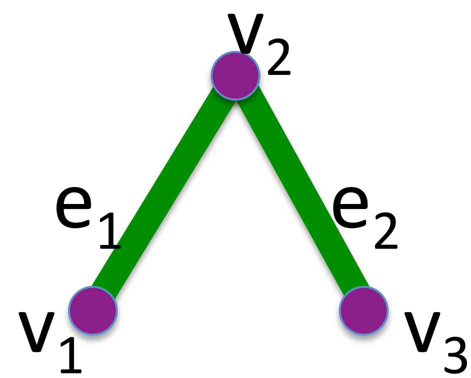
$$\begin{array}{c|c} & \{v_4, v_5, v_6\} \\ \hline \{v_1, v_2\} & 0 \\ \{v_2, v_3\} & 0 \\ \{v_4, v_5\} & 1 \\ \{v_5, v_6\} & 1 \\ \{v_4, v_6\} & 1 \end{array}$$

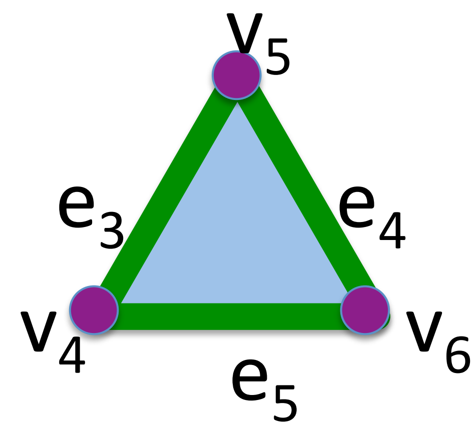


B_1 = image of ∂_2 = column space of M_2

$$C_2 \xrightarrow{\partial_2} C_1 \xrightarrow{\partial_1} C_0$$

$$\begin{array}{c} \\ \{v_1, v_2\} \\ \{v_2, v_3\} \\ \{v_4, v_5\} \\ \{v_5, v_6\} \\ \{v_4, v_6\} \end{array} \begin{array}{c} \{v_4, v_5, v_6\} \\ \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} \end{array}$$

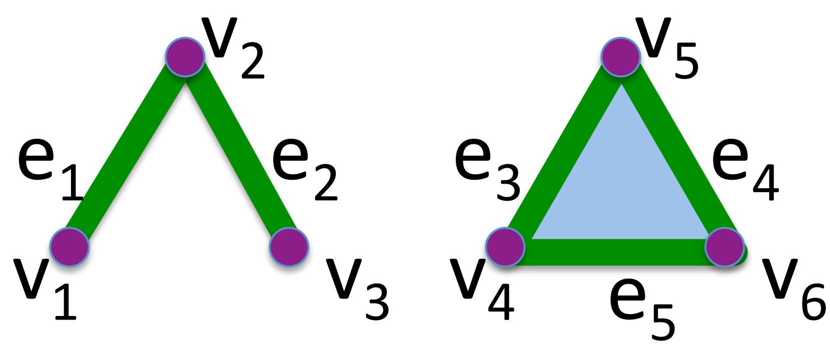




B_1 = image of ∂_2 = column space of M_2

= $<\{v_4, v_5\} + \{v_5, v_6\} + \{v_4, v_6\}>$ = $<e_3 + e_4 + e_5>$

$$C_2 \xrightarrow{\partial_2} C_1 \xrightarrow{\partial_1} C_0$$



$$H_1 = Z_1/B_1 = (\text{kernel of } \partial_1)/(\text{image of } \partial_2)$$

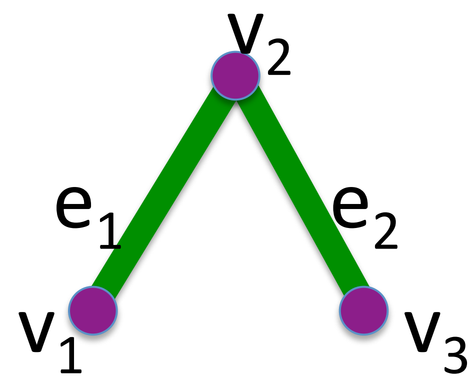
$$= \frac{\text{null space of } M_1}{\text{column space of } M_2}$$

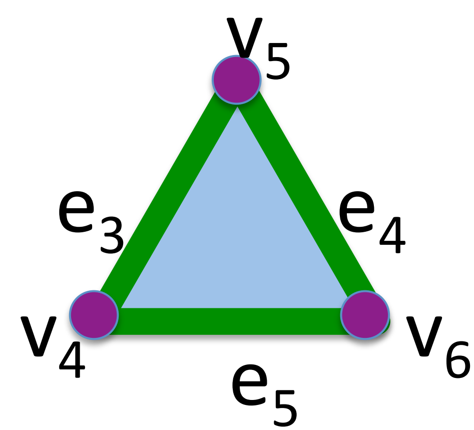
$$= \frac{\langle e_3 + e_4 + e_5 \rangle}{\langle e_3 + e_4 + e_5 \rangle}$$

$$\text{Rank } H_1 = \text{Rank } Z_1 - \text{Rank } B_1 = 1 - 1 = 0$$

$$C_2 \xrightarrow{\partial_2} C_1 \xrightarrow{\partial_1} C_0$$

$$\begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{array} \begin{array}{c} \{v_4, v_5, v_6\} \\ \left(\begin{array}{c} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{array}\right) \end{array}$$

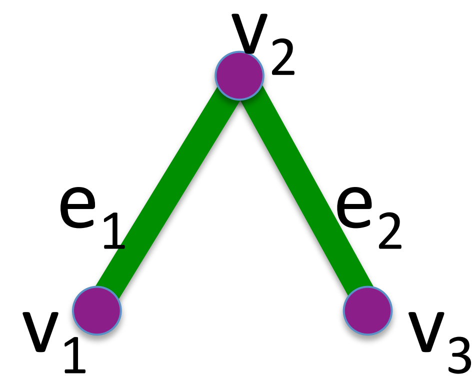




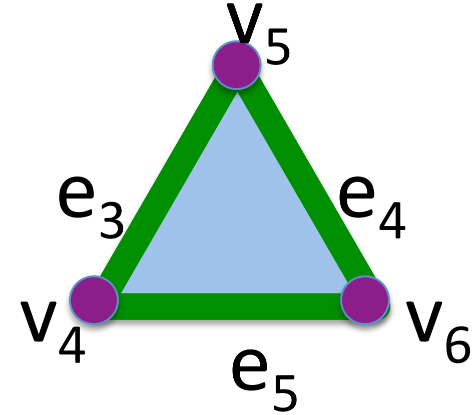
B_1 = image of ∂_2 = column space of M_2

$= \langle \{v_4, v_5\} + \{v_5, v_6\} + \{v_4, v_6\} \rangle = \langle e_3 + e_4 + e_5 \rangle$

$$C_2 \xrightarrow{\partial_2} C_1 \xrightarrow{\partial_1} C_0$$



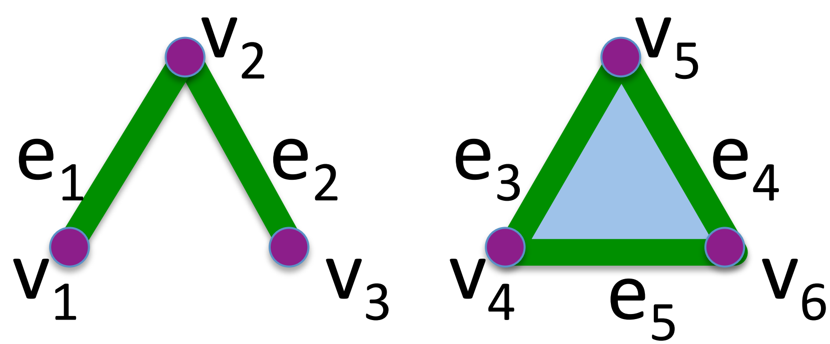
$$\begin{array}{r} \\ e_1 \\ e_2 \\ e_3 + e_4 + e_5 \\ e_4 \\ e_5 \end{array} \begin{array}{c} \{v_4, v_5, v_6\} \\ \left(\begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{array} \right) \end{array}$$



B_1 = image of ∂_2 = column space of M_2

$= \langle\{v_4, v_5\} + \{v_5, v_6\} + \{v_4, v_6\}\rangle = \langle e_3 + e_4 + e_5\rangle$

$$C_2 \xrightarrow{\partial_2} C_1 \xrightarrow{\partial_1} C_0$$



$$H_1 = Z_1/B_1 = (\text{kernel of } \partial_1)/(\text{image of } \partial_2)$$

$$= \frac{\text{null space of } M_1}{\text{column space of } M_2}$$

$$= \frac{\langle e_3 + e_4 + e_5 \rangle}{\langle e_3 + e_4 + e_5 \rangle}$$

$$\text{Rank } H_1 = \text{Rank } Z_1 - \text{Rank } B_1 = 1 - 1 = 0$$

http://www.ima.umn.edu/2008-2009/ND6.15-26.09

IMA

New Directions Short Course
Applied Algebraic Topology

June 15-26, 2009

Tuesday June 16, 2009

11:00am-12:30pm	"Homology 2" morse, morse-conley, hodge & more: simple applications	**Robert Ghrist** (University of Pennsylvania)

"Homology 2" morse, morse-conley, hodge & more: simple applications

June 16, 2009 11:00 am - 12:30 pm

- Lecture 4 slides (pdf)
- titlepage.pdf (pdf)
- Video (flv)

http://www.ima.umn.edu/2008-2009/ND6.15-26.09/abstracts.html#8322