NAME (PRINT): ________________________________

I pledge to NOT disclose the content of this exam to anyone (SIGN BELOW):

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MATH 116: Introduction to Complex Analysis
Midterm I, Spring 2014

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Rules of the exam

- You have 50 minutes to complete this exam.
- Show your work! – any answer without an explanation will get you zero points.
- Please read the questions carefully; some ask for more than one thing.
- When applicable, BOX the answer.
- Do not forget to write your name.

Good luck!
PROBLEM 1: (25 points) Define each of the terms listed below:

1. The Cauchy Riemann equations
2. a formula for computing the radius of convergence for a power series
3. rectifiable path
4. function \( \gamma : [a, b] \to \mathbb{C} \) of bounded variation
5. Leibniz’s rule
6. Let \( z_2, z_3, z_4 \) be points in \( \mathbb{C}_\infty \). Define the Mobius map \( S : \mathbb{C}_\infty \to \mathbb{C}_\infty \) such that \( S(z_2) = 1 \), \( S(z_3) = 0 \), \( S(z_4) = \infty \).
PROBLEM 2: (25 points) State and prove Morera’s Theorem.
PROBLEM 3: (20 points) State and prove Casorati-Weierstrass Theorem.
PROBLEM 4: (20 points) Solve at your choice ONE of the following problems.

1) Let $G$ be a region and suppose that $f : G \rightarrow \mathbb{C}$ is analytic such that $f(G)$ is a subset of a circle. Show that it is constant.

2) Let $f$ be an entire function and suppose that its range omits a disk $B(a, r)$ with $r > 0$. Show that $f$ is constant.
PROBLEM 5: (10 points) Let $h : \mathbb{C} \to \mathbb{R}$ be a non-constant, harmonic function. Then show that for every $y \in \mathbb{R}$ the preimage $f^{-1}(\{y\})$ is a nonempty, closed, unbounded subset of $\mathbb{C}$. 
BEAUTIFUL PROBLEM : (5 points) Assume that $T$ is a Mobius transformation which flips to given distinct points in the plane. What can you say about $T$?