SYLLABUS Fall 2017
The University of Iowa
The College of Liberal Arts and Sciences
Department of Mathematics
Functional Analysis (MATH:7200-0001)
1:30-2:20am /MWF  105 MLH (Lecture)

Website address: http://homepage.divms.uiowa.edu/~ichifan/teaching10.html

Some of the policies relating to this course (such as the drop deadline) are governed by its administrative home, the College of Liberal Arts and Sciences, 120 Schaeffer Hall

Instructor:  Ionut Chifan  (yo-nüts key-fun)
Office location:  1R MLH
Office hours:  1R MLH, M 2:30-3:20pm, W 10:30-11:20am, F 9:30-10:20am and by appointment
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DEO Contact Information:
Professor Maggy Tomova, 14 MLH, maggy-tomova@uiowa.edu

Prerequisite: MATH: 6210

Description of Course: This will be an introductory two-semesters course on various topics in modern ergodic theory of group actions on probability spaces. The course will often emphasize the intimate connections between this subject, representation theory, functional analysis, and von Neumann algebras. The main purpose is to provide the fundamentals needed to start active research in these fields. First semester covers the basic facts in the theory of group representations and group actions on probability spaces: basic properties of Hilbert spaces; compact operators and trace class operators; functions of positive type; cocycles; induced representations; weak containment; mixing properties; approximation properties (amenability, property T, etc ); orbit equivalence; Ornstein-Weiss theorem, Popa’s cocycle superrigidity theorem for Bernoulli actions; Ioana’s cocycle superrigidity theorem for profinite actions; other additional topics if time permits. The second semester covers other specific topics in contemporary ergodic theory, including sofic groups; Bowen’s sofic entropy; multiple recurrence, etc.

Objectives and Goals of the Course: This two semesters course is intended to familiarize the student with a new trend in functional analysis, specifically the classification of equivalence relations arising from actions of various countable groups on probability measure spaces. Our main goal is to present a self-contained approach towards natural examples of measure preserving OE-superrigid actions of countable groups on probability spaces (i.e. that can be completely reconstructed from their orbits). For instance we will show this is the case for all Bernoulli or profinite actions of icc property T groups
(Popa ’06, Ioana ’08). These results have been conjectured to exist for a long time (some say even from the beginning of the theory in early 40’s) but the first concrete examples emerged only over in the last two decades. The methods involved in proving this type of results are extremely diverse, emerging from many areas of math, but in this class I will focus on the measured group theory/functional analysis/von Neumann algebras aspects. On a different direction, in the second semester, we will study other aspects of measure preserving actions including Bowen’s sofic entropy theory and multiple recurrence. These are rapid developing areas of Mathematics with plenty of opportunities for graduate students to do active and rewarding research.

Texts:
For my lectures I will use other online notes that can be found at the following sources:

https://perso.univ-rennes1.fr/bachir.bekka/KazhdanTotal.pdf


The course will cover the following basic topics in group representations/group actions/orbit equivalence/ (using the above online lecture notes):

1. Basic properties of Hilbert spaces
2. Representations of groups
   a. Definitions, constructions, examples
   b. Functions of positive type
   c. Cocycles
   d. Induced representations
   e. Invariant vectors
   f. Weak containment
   g. Mixing properties
   h. Amenability
   i. Kazhdan’s property (T)
3. Compact operators and trace class operators;
4. Groups actions on measure spaces and their extensions
   a. Examples
   b. Koopman representation
   c. Ergodicity, weak mixing, compactness
   d. Ergodic theorems
5. Orbit Equivalence
b. Zimmer’s cocycle

c. Popa’s cocycle superrigidity thm for Bernoulli actions of property T groups (2006)

d. Ioana’s cocycle superrigidity thm for profinite actions of property T groups (2008)

**Grading System and the Use of +/-:** I will use the +/- grading system. You will be graded according to the following scale:

A [100,93); A- [90,93)
B+ [87,90); B [83,87); B- [80,83)
C+ [77,80); C [73,77); C- [70,73)
D+ [67,70); D [63,67); D- [40,63)
F [0, 40)

The grade of A+ will be awarded in extremely rare circumstances only for truly exceptional performance in the class.

**Assignments and Percentage of Final Grade:**
Presentation: 100%

**A Word about the Date and Time of the Final Exam:** There will be no final exam in this class, the grade being entirely based on a presentation close to the end of the semester. As the semester progresses he instructor the instructor will assign to each student a topic for presentation.

**Calendar of Course Assignments and Exams**

<table>
<thead>
<tr>
<th>Week</th>
<th>Beg-End</th>
<th>No lectures</th>
<th>Sections covered</th>
<th>Activities</th>
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<tbody>
<tr>
<td>1</td>
<td>8/21 - 8/25</td>
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<td>1, 2a</td>
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<td>2</td>
<td>8/28 - 9/1</td>
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<td>2b, 2c</td>
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<td>9/6 - 9/8</td>
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<td>9/11 - 9/15</td>
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<td>10/2 - 10/6</td>
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<td>14</td>
<td>11/27 - 12/1</td>
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<td>15</td>
<td>12/4 - 12/8</td>
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<td>Presentations</td>
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**Resources for Students:**
Students will find the Writing Center and the Speaking Center very useful for this course:

- **Writing Center:** [http://www.uiowa.edu/~writingc/](http://www.uiowa.edu/~writingc/)
- **Speaking Center:** [http://clas.uiowa.edu/rhetoric/for-students/speaking-center](http://clas.uiowa.edu/rhetoric/for-students/speaking-center)
- **Math Tutorial Lab:** 125 MLH [http://www.math.uiowa.edu/math-tutorial-lab](http://www.math.uiowa.edu/math-tutorial-lab)

**Course Policies:**

*Course attendance:* Attendance is expected for each class meeting, as it will help you better understand the concepts covered in lectures. If you miss a class, you are responsible for any assignments/announcements made/material covered.

*Participation in class discussions:* I strongly encourage you to actively participate in class discussions; ask questions or ask for more explanations whenever you feel confused; in this class there is NO stupid question!

*Final Presentation:* As the semester progresses the instructor will make available a number of topics from which the students can make their final presentations. Three weeks before the semester ends the students should meet with the instructor letting him know what and when they want to present.

**Student Collaboration:**
The homework for this course is designed to help you master your knowledge related to the topics covered during lecture. As such, you may work on the homework problems with others or use online resources. However, please be aware that to master the skills needed for this class, practice is required and that to do well on the final exam you will need to work many of these problems multiple times without help. Be sure to test your knowledge by doing much of the homework on your own. Even if you collaborate with your colleagues when solving the homework problems, I strongly encourage you not to copy, mot-a-mot, the solutions from others but instead try to write them in your own understanding. This is an excellent exercise proven to help students with their material comprehension.

**Other Expectations of Student Performance**

*Cell phones policy:* I am expecting you to NOT use your cell phones, i-pads, or computers during the lecture time for other purposes than class related.

*Complaint procedure:* Any student having a problem with the course should contact the instructor. Most issues can be resolved with a straightforward discussion.

*Changing grade policy:* If I change your grade on a homework or exam you should always remind me in the same day by e-mail that I have changed your grade.

Teaching Policies & Resources — Syllabus Insert
Administrative Home
The College of Liberal Arts and Sciences is the administrative home of this course and governs matters such as the add/drop deadlines, the second-grade-only option, and other related issues. Different colleges may have different policies. Questions may be addressed to 120 Schaeffer Hall, or see the CLAS Academic Policies Handbook at https://clas.uiowa.edu/students/handbook.

Electronic Communication
University policy specifies that students are responsible for all official correspondences sent to their University of Iowa e-mail address (@uiowa.edu). Faculty and students should use this account for correspondences (Operations Manual, III.15.2, k.11).

Accommodations for Disabilities
The University of Iowa is committed to providing an educational experience that is accessible to all students. A student may request academic accommodations for a disability (which includes but is not limited to mental health, attention, learning, vision, and physical or health-related conditions). A student seeking academic accommodations should first register with Student Disability Services and then meet with the course instructor privately in the instructor's office to make particular arrangements. Reasonable accommodations are established through an interactive process between the student, instructor, and SDS. See https://sds.studentlife.uiowa.edu/ for information.

Nondiscrimination in the Classroom
The University of Iowa is committed to making the classroom a respectful and inclusive space for all people irrespective of their gender, sexual, racial, religious or other identities. Toward this goal, students are invited to optionally share their preferred names and pronouns with their instructors and classmates. The University of Iowa prohibits discrimination and harassment against individuals on the basis of race, class, gender, sexual orientation, national origin, and other identity categories set forth in the University’s Human Rights policy. For more information, contact the Office of Equal Opportunity and Diversity, diversity@uiowa.edu, or visit diversity.uiowa.edu.

Academic Honesty
All CLAS students or students taking classes offered by CLAS have, in essence, agreed to the College's Code of Academic Honesty: "I pledge to do my own academic work and to excel to the best of my abilities, upholding the IOWA Challenge. I promise not to lie about my academic work, to cheat, or to steal the words or ideas of others; nor will I help fellow students to violate the Code of Academic Honesty." Any student committing academic misconduct is reported to the College and placed on disciplinary probation or may be suspended or expelled (CLAS Academic Policies Handbook).

CLAS Final Examination Policies
The final examination schedule for each class is announced by the Registrar generally by the fifth week of classes. Final exams are offered only during the official final examination period. No exams of any kind are allowed during the last week of classes. All students should plan on being at the UI through the final examination period. Once the Registrar has announced the date, time, and location of each final exam, the complete schedule will be published on the Registrar’s web site and will be shared with instructors and students. It is the student's responsibility to know the date, time, and place of a final exam.

Making a Suggestion or a Complaint
Students with a suggestion or complaint should first visit with the instructor (and the course supervisor), and then with the departmental DEO. Complaints must be made within six months of the incident (CLAS Academic Policies Handbook).

Understanding Sexual Harassment
Sexual harassment subverts the mission of the University and threatens the well-being of students, faculty, and staff. All members of the UI community have a responsibility to uphold this mission and to contribute to a safe environment that enhances learning. Incidents of sexual harassment should be reported immediately. See the UI Office of the Sexual Misconduct Response Coordinator for assistance, definitions, and the full University policy.

Reacting Safely to Severe Weather
In severe weather, class members should seek appropriate shelter immediately, leaving the classroom if necessary. The class will continue if possible when the event is over. For more information on Hawk Alert and the siren warning system, visit the Department of Public Safety website.