

Math 16, Final Review Questions
May, 2003

- (1) Work through the first two review sheets as well as the first two exams.
- (2) Give the definition of the integral $\int_a^b f(x) dx$ as a limit of sums. Make sure that any notation which you use is explained. That is, when you write $\sum_{j=1}^n f(x_j)\Delta x_j$, make sure you explain what the x_j are and what Δx_j means. Draw a picture illustrating the meaning of the sum and explain what your picture has to do with the sum.
- (3) What is an antiderivative of a function? Give an example illustrating the concept.
- (4) State the fundamental theorem of calculus (exactly as it was stated in class.) Explain how the FTC is used to calculate integrals and give an example of such a computation.
- (5) Derive the formula for integration by substitution, starting with the chain rule for derivatives. Give an typical example for which this technique of integration is useful.
- (6) Derive the formula from integration by parts, starting with the product rule for derivatives. Give an typical example for which this technique of integration is useful.
- (7) What are the formulas for $\sin^2(x)$ and $\cos^2(x)$ in terms of $\cos(2x)$? How can these formulas be used to calculate $\int \sin^2(x) dx$ or $\int \cos^2(x) dx$?
- (8) The following differential equation appeared on the second midterm:

$$\frac{dy}{dx} = y^2.$$

Use the technique developed in class on Friday May 2 to find the general solution to this differential equation.

- (9) Calculate the following antiderivatives or definite integrals. As always, check your antiderivatives by taking the derivative of your proposed answer.
 - (a) $\int x^3 - 3x^5 + \sqrt{x} - e^x dx$
 - (b) $\int \cos(2x) - 5 \sin(3x) dx$
 - (c) $\int \tan(x) dx$
 - (d) $\int \ln(x) dx$

(e) $\int \sin^3(x) \cos(x) dx$

(f) $\int \sin^3(x) dx$

(g) $\int x \sin(x) dx$

(h) $\int x^2 \sin(x) dx$

(i) $\int \frac{x^3}{x^4 + 2} dx$

(j) $\int \frac{x^3}{\sqrt{x^4 + 2}} dx$

(k) $\int_0^{\pi/4} e^{\cos(x)} \sin(x) dx$

(l) $\int_0^{\pi/4} e^{\cos(x)} \sin(x) dx$

(m) $\int e^{x^2} x dx$

(n) $\int e^{x^2} x^2 dx$. (Integration by parts!)

- (10) Find the area between the curve $y = x^3 + 1$ and the x -axis between $x = 1$ and $x = 2$.
- (11) Find the area between the two curves $y = x + 2$ and $y = x^2$.
- (12) What is the area between the x -axis and the curve $y = 1/x$ between $x = 1$ and $x = e$?
- (13) A medication is injected intramuscularly. The amount of medicine in the blood at time t hours after the injection is

$$B(t) = 75(e^{-.8t} - e^{-.5t}).$$

Graph $y = B(t)$. Compute the bioavailability of the medication during the first 24 hours after the injection.

- (14) Calculate the volume of a sphere of radius R by computing an appropriate integral.