Math 16, First Midterm Review Questions
February 26, 2003

(1) What is the definition of the derivative of a function?

(2) What are the various interpretations of the notion of the derivative of a function?

(3) Compute the derivatives of the following functions *directly from the definition of the derivative.* That is, first write down

$$\frac{f(x+h)-f(x)}{h}$$

and then manipulate the expression until the limit as $h \to 0$ can be determined by inspection.
(a) $f(x) = x^2$.
(b) $f(x) = 1/x^2$
(c) $f(x) = \sqrt{x}$
(d) $f(x) = 7$.

(4) A bacterial population grows according to the law:

$$P(t) = P(0)(1.3)^t,$$

where t is measured in hours.

(a) At what time will the population be three times the initial value $P(0)$?
(b) At what time will the population be six times the initial value $P(0)$?
(c) What is the derivative of $P(t)$?
(d) At what rate is the population changing when t = 2 hours?

(5) Suppose that f and g are two functions whose derivatives are know. What are the rules for evaluating the derivatives of
(a) $f(x)g(x)$
(b) $1/f(x)$
(c) $f(x)/g(x)$
(d) $\ln(f(x))$
(e) $f \circ g(x) = f(g(x))$
(f) $(f(x))^5$
(g) $13f(x)$.

(6) Find the tangent line to the graph of $y = \sqrt{x}$ at $x = 3$.

(7) Find the composition inverse of the function $f(x) = \dfrac{x}{1+x}$. That is, find a function $g(x)$ such that $f(g(x)) = x$.

(8) Let $f(x) = \dfrac{x}{1+x}$ and $g(x) = x^2 + 2$. Find $f \circ g$ and $g \circ f$. Compute the derivatives of $f \circ g$ and of $g \circ f$.

(9) Compute the derivatives in the two practice sheets which I have provided.

(10) Compute the derivatives in "focus on practice" section at the end of Chapter 3. Remember that you can check your work using Mathematica, or by using the web site to which I directed you.

(11) Study the Review problems for chapter 3 on page 157 of the text.

(12) Assume that the temperature in Iowa City during a certain period in January follows a sinusoidal daily oscillation, with average value 25 degrees F., amplitude 10 degrees F., period 24 hours (of course), and the daily maximum temperature occuring at 1 pm. Find a formula for the temperature as a function of time t, where t is measured in hours, and $t = 0$ represents midnight of the first day of the period during which the description is valid. Draw a graph of the temperature function. Hint: The function will have the form $f(t) = 25 + A\sin(k(t+\delta))$. You have to adjust A, k, and δ in order to fit all the requirements of the problem.

(13) Draw a possible graph of a function based on the following information:
  (a) $f'(x)$ is positive for $x < 3$.
  (b) $f'(x)$ is negative for $x > 3$.
  (c) $f(0) = 0$
  (d) $f(x) > 0$ for $x > 0$.
  (e) $f''(x) < 0$ for $x < 5$.
  (f) $f''(x) > 0$ for $x > 5$.

(14) (a) A potato at room temperature is put into an oven at 350 degrees F. Based on common sense, draw a graph representing the temperature of the potato as a function of time.

What would you surmise about the slope and the convexity of the graph?

(b) Compare your common sense conclusions with Exercise 48 on page 158 of the text.

(15) Consider the function $f(x) = \sin(x^4)$. Since the sin function oscilates between -1 and $+1$, so does $f(x)$. At what values of x does the function $f(x)$ attain its maximum value of $+1$. (Hint: there are infinitely many such x.) What is the derivative of $f(x)$? Does the derivative also oscilate between fixed values, or does it take arbitrarily large values? Make a rough sketch of the derivative function.

(16) A certain population follows the so called Monod growth law, according to which the population is equal to

$$P = \frac{cr}{k + r},$$

where c and k are fixed positive parameters, and r represents the available quantity of some resource. Suppose the resource r satisfies

$$r(t) = \sin(t) \ln(t),$$

for $t > 0$. What is $\frac{dP}{dt}$?