Math 16, Second Midterm Exam
April 20, 2000

Instructions: **Write your name on the top of this page now!**

This exam has 7 questions and 8 pages. The relative weight of the various questions is indicated. Do all the exercises, writing your answers in this exam booklet. Show your work; make sure all of your steps are justifiable and easy to follow. Your work will be judged for correctness, completeness, clarity and orderliness. Put your final answer to each question in a box so that it can be located easily.

1. (20 points)

    (a) What is the definition of the *derivative* of a function?

    (b) Compute the derivative of $f(x) = \dfrac{1}{x}$ *directly from the defintion of the derivative.* (Make sure that your answer agrees with that obtained by using appropriate rules for differentiation.)

2. (15 points) Find the equation of the tangent line to $y = \sqrt{x}$ at the point $(4, 2)$.

3. (25 points) Compute the derivatives of the following functions:

(a) $f(x) = \dfrac{x^2 - 4}{x^2 + 4}$.

(b) $f(x) = e^x(x^{13} + 2x^2)$.

(c) $f(x) = \cos(x)\ln(x)$.

4. (25 points) Compute the derivatives of the following functions:

    (a) $f(x) = \sqrt{x^2 + 1}$.

    (b) $f(x) = \sin(x^2)\sin^2(x)$.

    (c) $f(x) = e^{\sqrt{x^2+1}}$.

5. (25 points) Analyze the function
$$f(x) = xe^{-x^2}.$$
Find the intervals on which the function is increasing or decreasing. Find the intervals on which the the function is concave up or concave down. Find the limits of the function at $\pm\infty$. Find all local maxima and minima and all points of inflection. Use the information which you have found to sketch the graph of the function. If you have a graphing calculator, make sure your graph agrees with that shown by the calculator.

6. (25 points) Consider the function
$$f(x) = 4x^3 - 21x^2 + 10x + 2.$$

(a) Find the local maxima and minima of f; i.e. find both the x-values and the y-values at the local maxima and minima. Use this to make a rough sketch of the graph of f.

(b) Use the Step (a) to estimate the zeroes of f, i.e. to locate the zeroes approximately.

(c) Give the formula for a single step of Newton's method; i.e. given a function f, and a "guess" a for a zero of f, what is the Newton method formula for the "next guess."

(d) Use Newton's method to find *one* of the zeroes of
$$f(x) = 4x^3 - 21x^2 + 10x + 2$$
to two digits accuracy.

7. (25 points) Dr. Stanley Poisson of the National Bureau of Fisheries in Tiger Woods, Virginia, proposes the following population model for the Caribean Zebra Shrimp:

$$N_{t+1} = 2N_t e^{-N_t^2/4} - (1/2)N_t.$$

   (a) What is the updating function of the Poisson model?
   (b) Find the (non-negative) equilibrium points of the Poisson model.
   (c) Classify the equilibrium point(s) as stable or unstable.