Mathematics 150 Assignment 10 – F. Goodman December, 2007

Use generating function models to solve the following exercises:

- 1. Find the number of ways to select 25 balls from a pile of red, white and blue balls if:
 - (a) The selection has at least two red and at least two white balls.
 - (b) The selection has at most 3 red balls.
 - (c) The selection has an even number of blue balls.
- 2. How many ways can we order lunch for the 12 people in our class if their are 5 types of sandwiches available, but at most 4 sandwiches of any one type are available?
- **3.** How many ways are there to distribute 1 dollar (in pennies) to 10 children and one adult, if the adult receives either 5 cents or 10 cents and each child receives at most 15 cents?
- 4. A coin is flipped 25 times, and exactly 8 tails occur. In how many ways can this be done (i.e. how many sequence of 25 H's and T's with 8 T's). In how many ways can this be done so that there are no runs of 6 or more heads?
- 5. A probability generating function $p_X(t)$ for a discrete random variable X is a power series $\sum_r p_r t^r$, where p_r is the probability that the random variable takes the value r.
 - (a) Let X be the number of heads that occur when a fair coin is flipped n times. Find $p_X(t)$.
 - (b) Let X be the number of times a fair coin is flipped until the 5th head occurs. Find $p_X(t)$.