Mathematics 120 Midterm Exam II – F. Goodman November , 2005 Version 1

Responses will be judged for accuracy, clarity and coherence.

- 1. Show that if a finite group G acts on a finite set X, then the size of every orbit divides the size of the group.
- 2. Show that a group of order p^n , where p is a prime, has a non-trivial center.
- 3. State the structure theorem for finite abelian groups in both the invariant factor form and the elementary divisor form. List all abelian groups of order $216 = 2^3 3^3$; give both the invariant factor decomposition and the elmentary divisor decompositon of each group.
- 4. Prove Cauchy's theorem (regarding the existence of subgroups of prime order).
- 5. State the complete Sylow Theorem (all three parts). I don't care about numbering the parts.