CONFERENCE ON GEOMETRIC METHODS IN REPRESENTATION THEORY

UNIVERSITY OF IOWA – IOWA CITY

NOVEMBER 18-20, 2017

Conference Schedule

All talks are in MACLEAN HALL, rooms MacLean 110 and MacLean 118.

	Saturday, November 18
8:45-9:00	WELCOME (118 MLH)
9:00-10:00	**Pevtsova** (keynote) (118 MLH) *Detection of nilpotents in cohomology and projectivity of modules for finite (super) group schemes.*
10:00-10:10	Questions
10:10-10:40	Coffee
10:40-11:30	**Derksen** (expository) (118 MLH) *Stability of quiver representations.*
11:30-11:45	Questions
11:45-12:15	**Weyman** (118 MLH) *Finite free resolutions and root systems.*
12:15-12:25	Questions
12:25-2:30	LUNCH BREAK
2:30-3:00	**Chinburg** (118 MLH) *Congruences between modular forms via representation theory.*
3:00-3:10	Questions
3:10-4:00	Coffee **REGISTRATION** (cash or check)
4:00-4:30	**Igusa** (118 MLH) *Are finite type picture groups virtually special?*
4:30-4:45	Questions and move to parallel talks
4:45-5:05	**Lindo** (118 MLH) *Trace modules and rigidity.* **Cheung** (110 MLH) *Quiver representations and theta functions.*
5:05-5:15	Questions

	Sunday, November 19
9:00-10:00	**Pevtsova** (keynote) (118 MLH) *Detection of nilpotents in cohomology and projectivity of modules for finite (super) group schemes.*
10:00-10:10	Questions
10:10-10:40	Coffee
10:40-11:30	**Yakimov** (expository) (118 MLH) *The role of Poisson orders in representation theory.*
11:30-11:45	Questions
11:45-12:15	**Carlson** (118 MLH) *Virtual projectivity and p-divisible modules.*
12:15-12:25	Questions
12:25-2:30	LUNCH BREAK
2:30-3:00	**Dao** (118 MLH) *Canonical cover and non-commutative desingularizations.*
3:00-3:10	Questions
3:10-3:30	Coffee
3:30-4:00	**Todorov** (118 MLH) *Fin.dim. and phi-dim.*
4:00-4:15	Questions and move to parallel talks
4:15-4:35	**Beil** (118 MLH) *A first look at homotopy dimer algebras on surfaces with boundary.* **Meyer** (110 MLH) *An isometry theorem for incidence algebras.*
4:35-4:45	Questions

	Monday, November 20
9:00-9:30	**Lin** (118 MLH) *Variety of semi-conformal vectors in a vertex operator algebra.*
9:30-9:40	Questions
9:40-10:00	**Weist** (118 MLH) *Tree normal forms for quiver representations.*
10:00-10:10	Questions
10:10-10:40	Coffee and move to parallel talks
10:40-11:00	**Kulkarni** (118 MLH) *Dimer models on cylinders over Dynkin diagrams and Cluster algebras.* **Weigandt** (110 MLH) *Partition identities and quiver representations.*
11:00:11:10	Questions
11:10-11:30	**Rupel** (118 MLH) *On Kontsevich automorphisms and quiver Grassmannians.* **Sistko** (110 MLH) *Maximal subalgebras of finite-dimensional algebras and applications.*
11:30-11:40	Questions
11:40-12:00	**Zhu** (118 MLH) *Auslander's formula in contravariantly finite subcategories of dualizing varieties.* **Wackwitz** (110 MLH) *Versal deformation rings and symmetric special biserial algebras.*
12:00-12:10	Questions
12:10-12:30	**Sen** (118 MLH) *Singularities of dual varieties associated to exterior representations.* **Vélez-Marulanda** (110 MLH) *Derived tame Nakayama algebras.*
12:30-12:40	Questions

Abstracts

Keynote Lectures

Julia Pevtsova (University of Washington).
Detection of nilpotents in cohomology and projectivity of modules for finite (super) group schemes.

For a finite group G, classical theorems of Quillen and Chouinard tell us how to detect whether a cohomology class in mod p cohomology is nilpotent or whether a module is projective: one has to restrict to elementary abelian subgroups of G. For connected finite group schemes, the detecting family consists of one-parameter subgroups as shown by Suslin, Friedlander, and Bendel. In these talks I'll review the history of the subject starting with Quillen and then will describe what plays the role of elementary abelian subgroups for finite unipotent super group schemes. Some interesting new phenomena arise when one introduces grading; the theory of Dieudonne modules plays an important role.

Based on joint work with D. Benson, S. Iyengar, and H. Krause.

Expository Lectures

Harm Derksen (University of Michigan).
Stability of quiver representations.

A weight for a quiver is an integer-valued function on the set of vertices. Alistair King systematically studied geometric invariant theory for quiver representations. He defined σ-semi-stability for any weight σ. I will first discuss known results about σ-semi-stability, and discuss some applications such as the representation theory for the general linear group, algebraic complexity theory and integral inequalities.

Milen Yakimov (Louisiana State University).
The role of Poisson orders in representation theory.

Many problems in representation theory concern the study of irreducible representations of algebras which are module-finite over their centers (restricted universal enveloping algebras in prime characteristic, quantum groups at roots of unity, PI Sklyanin algebras). The notion of Poisson orders brings strong Poisson geometric methods to these problems. We will review this setting, the Brown-Gordon theorem for isomorphisms across symplectic leaves, its applications to representation theory, relations to specializations and noncommutative discriminants. The last part of the talk is on a joint work with Bach Nguyen, Kurt Trampel, Chelsea Walton and Xingting Wang.

Charlie Beil (University of Bristol).
A first look at homotopy dimer algebras on surfaces with boundary.

I will report on work in progress with Karin Baur on a class of quiver algebras called homotopy algebras. These algebras coincide with dimer algebras when they are noetherian, and their quivers embed into (possibly singular) compact surfaces. I will present our main conjecture on homological smoothness, and give an explicit example where the conjecture holds.

Jon Carlson (University of Georgia).
Virtual projectivity and p-divisible modules.

Assume that G is a finite group and that k is a field of characteristic $p > 0$. The class of finitely generated modules, all of whose direct summands (after any field extension) have dimension divisible by p, is close under tensor product. Yet, modules in the class with full support variety still generated the entire stable category. There are good reasons for looking for subclasses of the modules. In joint work with Dave Benson, we investigate degrees of virtual projectivity as a stratifying method, obtaining some unexpected results.

Mandy Cheung (Harvard University).
Quiver representations and theta functions.

Scattering diagrams theta functions and broken lines were developed in order to describe toric degenerations of Calabi-Yau varieties and construct mirror pairs. Later, Gross-Hacking-Keel-Kontsevich unravel the relation of those objects with cluster algebras. In the talk, we will discuss how we can combine the representation theory with these objects. We will also see how the broken lines on scattering diagram give a stratification of quiver Grassmannians using this setting.

Ted Chinburg (University of Pennsylvania).
Congruences between modular forms via representation theory.

In this talk I will discuss some work with Frauke Bleher and Aristides Kontogeorgis. We determine the module structure of the holomorphic differentials of a finite G-cover of curves in odd characteristic p when the Sylow p-subgroups of G are cyclic. One application is to produce new congruences between weight two cusp forms for the principal congruence subgroup of level ℓ when ℓ is an odd prime different from p.

Hailong Dao (University of Kansas).
Canonical cover and non-commutative desingularizations.

Let R be a normal domain. In this work we first show that if R admits a noncommutative crepant resolution (NCCR) in the sense of Van den Bergh, then it is Q-Gorenstein (the canonical class is torsion in the class group). We then establish tight connections between the existence of noncommutative desingularizations on R and its canonical cover S. The results immediately give some corollaries: non-Gorenstein quotient singularities by connected

reductive groups cannot admit an NCCR, the centre of any NCCR has log-terminal singularities, and the Auslander-Esnault classification of two-dimensional Cohen-Macaulay-finite algebras. This is joint work with Iyama, Takahashi and Wymess.

Kiyoshi Igusa (Brandeis University).
Are finite type picture groups virtually special?

This is a report on work of Eric Hanson, a beginning PhD student at Brandeis working under the direction of Corey Bergman and me. Eric is working on combinatorial group theory and representation theory using picture groups and special cube complexes. For Λ any finite dimensional algebra over any field, one can use τ-tilting theory to define the "picture group" $G(\Lambda)$ of Λ. This group is finitely generated if and only if Λ is τ-tilting finite. In the case Λ is hereditary of finite type, Gordana Todorov and I proved that the picture group is a "CAT(0)"-group by constructing a compact "NPC" cubical space with fundamental group equal to $G(\Lambda)$. If the cube complex satisfies additional conditions, as pioneered by Haglund and Wise, the space and group are called "special". Picture groups share many of the properties of special groups, so the idea was that they could be special. Eric says this is too optimistic. His conjecture is: "Picture groups of Dynkin quivers are virtually special". Even this weaker condition would imply very nice properties of the picture groups. For example, it would imply that these groups embed in $SL_n(\mathbb{Z})$. The purpose of this talk is to describe this project and some beginning steps made by Eric Hanson.

Maitreyee Kulkarni (Louisiana State University).
Dimer models on cylinders over Dynkin diagrams and Cluster algebras.

Let G be a Lie group of type ADE and let P be a parabolic subgroup. It is known that there exists a cluster structure on the coordinate ring of the partial flag variety G/P (see the work of Geiss, Leclerc, and Schröer). Since then there has been a great deal of activity towards categorifying these cluster algebras. Jensen, King, and Su gave a direct categorification of the cluster structure on the homogeneous coordinate ring for Grassmannians (that is, when G is of type A and P is a maximal parabolic subgroup). In this setting, Baur, King, and Marsh gave an interpretation of this categorification in terms of dimer models. In this talk, I will give an analog of dimer models for groups in other types by introducing a technique called constructing cylinders over Dynkin diagrams, which can (conjecturally) be used to generalize the result of Baur, King, and Marsh.

Zongzhu Lin (Kansas State University).
Variety of semi-conformal vectors in a vertex operator algebra.

A vertex operator algebra consists of a vertex algebra structure and compatible conformal structure, i.e., a module structure of Virasoro Lie algebra arising from a vertex operator. The vector defining this vertex operator is called the Virasoro vector. A vertex operator subalgebra can possibly have different conformal structure. In this talk I will talk on those vertex operator sugalgebras that has a different Virasoro vector that induces a modules structure for the half of the Virasoro Lie algebra. We will call such conformal element as semi-conformal element. The set of all semi-conformal vectors form an algebraic variety. The coset constructions of semi-conformal vertex operator subalgebras gives raise to another

semi-conformal vertex operator algebras. The properties of the semi-conformal vertex operator subalgebras under the coset construction completely characterize the Heisenberg vertex operator algebras. We will also describe the these varieties for Heisenberg vertex operator algebras and affine vertex operator algebras of type A1.

Haydee Lindo (Williams College).
Trace modules and rigidity.

We will discuss the relationship between trace modules and the vanishing of $\mathrm{Ext}^1_R(M, M)$ for modules M over Noetherian rings R. The findings will be applied to a conjecture by Auslander and Reiten in the context of self-injective commutative rings.

David Meyer (University of Missouri-Columbia).
An isometry theorem for incidence algebras.

In recent work, generalized persistence modules have proved useful in distinguishing noise from the legitimate topological features of a data set. If P is a finite poset, a generalized persistence modules for P with values in the category K-mod is exactly a module for the incidence algebra of P. In topological data analysis the interplay between various metrics on isomorphism classes of generalized persistence modules is of significant interest, most notably the isometry theorem of Bauer and Lesnick. In this talk I present an algebraic analogue of the isometry theorem for a certain class of finite posets.

Dylan Rupel (University of Notre Dame).
On Kontsevich automorphisms and quiver Grassmannians.

In this talk I will describe a combinatorial construction of non-commutative Laurent polynomials arising from iterations of polynomial Kontsevich automorphisms acting on the skew-field of rational functions in two non-commuting variables. By specializing to q-commuting variables and passing through the categorification of quantum cluster algebras, this combinatorics gives rise to a construction of the counting polynomials of quiver Grassmannians for valued quivers with two vertices. After this I will briefly discuss work in progress with Thorsten Weist toward understanding implications of these results on the geometry of quiver Grassmannians for n-Kronecker quivers.

Emre Sen (Northeastern University).
Singularities of dual varieties associated to exterior representations.

For a given irreducible projective variety X, the closure of the set of all hyperplanes containing tangents to X is the projectively dual variety $X^\vee$. We study the singular locus of projectively dual varieties of certain Segre-Plücker embeddings. We give a complete classification of the irreducible components of the singular locus of several representation classes. Basically, they admit two types of singularities: cusp type and node type which are degeneracies of a certain Hessian matrix, and the closure of the set of tangent planes having more than one critical point respectively. In particular, our results include a description of singularities of dual Grassmannian varieties.

Alex Sistko (University of Iowa).
Maximal subalgebras of finite-dimensional algebras and applications.

We present a classification for maximal subalgebras of finite-dimensional algebras over a field K. This is done by first classifying maximal subalgebras of semisimple algebras, and then lifting to the general case. When K is nice (ex. algebraically closed), the classification can be understood in terms of the ideal structure of the Jacobson radical. For bound quiver algebras, this gives us nice presentations for subalgebras. We will discuss the role of trivial and separable extensions in our classification, and the problem of determining isomorphism classes of maximal subalgebras.

Gordana Todorov (Northeastern University).
Fin.dim. and phi-dim.

I will recall the statement of the Finitistic Dimension Conjecture and some already known results. The notion of φ-function was originally introduced in order to deal with the above conjecture. Since φ-function at any finitely generated module is a finite integer and furthermore is a generalization of projective dimension, it can be considered as a new invariant for modules of infinite projective dimension. Globally, φ-dim of an algebra is defined as the supremum of $\varphi(M)$. I will discuss some old and some new results.

José Vélez-Marulanda (Valdosta State University).
Derived tame Nakayama algebras.

We determine derived representation type of Nakayama algebras and prove that a derived tame Nakayama algebra without simple projective modules is gentle or derived equivalent to some skewed-gentle algebra. As a consequence, we determine the singularity category of a derived tame Nakayama algebra without simple projective modules. This is joint-work with Viktor Bekkert and Hernán Giraldo.

Daniel Wackwitz (University of Wisconsin-Platteville).
Versal deformation rings and symmetric special biserial algebras.

Let k be an algebraically closed field of arbitrary characteristic. Suppose A is a symmetric special biserial algebra over k. For a finitely generated indecomposable A-module V, the versal deformation ring $R(A, V)$ of V is characterized by the property that every lift of V over a complete local commutative Noetherian k-algebra R with residue field k is, up to isomorphism, determined by a (not necessarily unique) local ring homomorphism from $R(A, V)$ to R. In this talk, I will present our progress on the classification of the versal deformation rings of indecomposable A-modules for symmetric special biserial algebras A, which focuses primarily on the case when A is of domestic representation type. This is joint work with David Meyer and Roberto Soto.

Anna Weigandt (University of Illinois).
Partition identities and quiver representations.

We present a particular connection between classical partition combinatorics and the theory of quiver representations. Specifically, we give a bijective proof of an analogue of A. L.

Cauchy's Durfee square identity to multipartitions. We then use this result to give a new proof of M. Reineke's quantum dilogarithm identities in type A.

Thorsten Weist (Bergische Universität Wuppertal).
Tree normal forms for quiver representations.

Motivated by a rather vague conjecture of Kac which says that the set of isomorphism classes of indecomposable quiver representations of a fixed dimension admits a cell decomposition into locally closed subvarieties which are affine cells, we discuss a concept which can be used to deduce normal forms for indecomposable representations of quivers based on the notion of tree modules. Actually, with a fixed tree module of a quiver and a fixed tree-shaped basis of its group of self-extensions, we can associate an affine space of representations with the same dimension vector. If all representations in this cell are indecomposable and pairwise non-isomorphic, this induces a normal form for these representations. This is for instance the case if the tree module is a torus fixed point under a certain torus action. But also recursive constructions of indecomposable representations turn out to be very useful. For certain roots, this machinery can be used to give a full description of all indecomposable representations up to isomorphism.

Jerzy Weyman (University of Connecticut).
Finite free resolutions and root systems.

In this talk I will discuss the structure of free resolutions of length 3 over Noetherian rings. Associate to a triple of ranks (r_3, r_2, r_1) in our free complex a triple $(p, q, r) = (r_3 + 1, r_2 - 1, r_1 + 1)$. Associate to (p, q, r) the graph $T_{p,q,r}$ (three arms of lengths $p - 1, q - 1, r - 1$ attached to the central vertex). The main result is the explicit construction of a generic ring R_{gen} for resolutions of the format with the differentials of ranks r_1, r_2, r_3. This ring deforms to a ring that carries an action of a Kac-Moody Lie algebra associated to the graph $T_{p,q,r}$. In particular the ring R_{gen} is Noetherian if and only if $T_{p.q.r}$ is a Dynkin graph. I will discuss the structure of the ring R_{gen} and possible consequences for the structure of perfect ideals of codimension 3.

Shijie Zhu (Northeastern University).
Auslander's formula in contravariantly finite subcategories of dualizing varieties.

Let Λ be an artin algebra. Auslander's formula shows a nice connection between the module category Λ– mod and the category of finitely presented functors: $\dfrac{(\Lambda\text{– mod})\text{– mod}}{\{F \mid F(\Lambda) = 0\}} \cong \Lambda\text{– mod}\,.$
Recently, it has been generalized by Asadollahi, Hafezi and Keshavarz to contravariantly finite subcategories and also generalized by Ogawa to dualizing varieties. We unify these two results and generalize the Auslander-formula to contravariantly finite subcategories of dualizing varieties.