Abstracts

Keynote Lectures

Gordana Todorov (Northeastern University).

Lecture 1: *Picture groups and Cluster theory.*
(Joint with K. Igusa, K. Orr, J. Weyman)

To each modulated quiver Q of finite Dynkin type we associate a "picture" defined using domains of semi-invariants (stability conditions). To each such picture we associate a group $G(Q)$, called "picture group." Also, to the quiver Q we associate a cubical category $C(Q)$, called "cluster morphism category," which is defined in terms of wide subcategories and cluster tilting objects. Denote the classifying space of the cluster morphism category by $X(Q)$. We show that the space $X(Q)$ is $K(\pi, 1)$, i.e. it has only one non-zero homotopy group π_1, and furthermore this group is isomorphic to the picture group $G(Q)$.

Lecture 2: *Picture groups and maximal green sequences.*
(Joint with K. Igusa, T. Brüstle, S. Hermes)

This talk will be about the use of properties of the domains of semi-invariants and geometry of this picture in order to study maximal green sequences. The notion of maximal green sequences comes from the theory of cluster algebras, and at the same time it also arises in some models of particle physics (source-sink mutations). Green mutation in a cluster algebra is defined as a mutation in the direction of a non-negative c-vector; this can also be interpreted as crossing domains of semi-invariants in a particular direction. We use this last property in order to deal with the open problems about sequences of green mutations. We have shown that maximal green sequences correspond to positive expressions of Coxeter in the picture group $G(Q)$ associated to the "picture" defined using domains of semi-invariants..

Conference Talks

Charlie Beil (University of Bristol).

Towards homological smoothness for dimer algebras.

A dimer algebra A is a type of quiver algebra whose quiver embeds in a torus, with homotopy-like relations. It is well known that if A has the cancellation property, then A is a noncommutative crepant resolution of its 3-dimensional singular center. In particular, A is noetherian and homologically smooth. If, in contrast, A is non-cancellative, then A is nonnoetherian with a nonnoetherian center. I will describe how such a center is also a 3-dimensional singularity, but with the strange property that it contains precisely one positive dimensional 'smeared-out' point. I will then describe how the homology of certain vertex simple representations reflects this nonlocal geometry, as well as generalizes the homological smoothness of cancellative dimer algebras.

Jon Carlson (University of Georgia).
Relative stable categories.

We will discuss the problem of classifying the thick subcategories of the stable categories of modules, modulo a collection of subgroups of a finite group. This problem is important to group representation theory because of the Green correspondence.

Andy Carroll (DePaul University).
Moduli spaces and Schur-tame algebras.

Attempts to generalize invariant-theoretic characterizations of representation type have met with some resistance. In the case of hereditary algebras, it was shown that an algebra is tame if and only if all moduli spaces of semi-stable points are products of projective spaces. Unfortunately, such a characterization doesnt hold for non-hereditary algebras. I will illustrate the failure of this characterization, and demonstrate preliminary results for Schur-tame algebras. This is joint work with Calin Chindris.

Ted Chinburg (University of Pennsylvania).
Invariants of quadratic forms.

In this talk I will begin by describing some results of Serre and Fröhlich in the 1980's which related Hasse-Witt invariants of the trace form associated to a finite Galois extension of number fields to Stiefel-Whitney classes. Many generalizations and applications of these results have been found, and by the end of the talk I will describe a few developed in joint work with Ph. Cassou-Noguès, B. Morin and M. J. Taylor.

Federico Galetto (Queen's University).
Equivariant resolutions of De Concini-Procesi ideals.

For modules over polynomial rings with a reasonable group action, the minimal free resolution of the module inherits an action by the same group. Understanding how the group acts on the resolution leads to a refinement of classical invariants of the module, such as the Betti numbers and the Hilbert series. In this talk, I will present examples of resolutions, with the action of a symmetric group, arising from certain ideals introduced by De Concini and Procesi with particular significance in geometry, combinatorics and representation theory.

Kiyoshi Igusa (Brandeis University).
Categories of noncrossing partitions.

In joint work with Gordana Todorov, we constructed a category whose morphisms are given by clusters. We call it the "cluster morphism category". This has applications to the cohomology of groups and to maximal green sequences. In a separate paper, I gave a combinatorial version of a special case of this category. Objects of this category are noncrossing partitions in the traditional sense. Hubery and Krause have also constructed a category of noncrossing partitions in which the objects are generalized noncrossing partitions. In this talk I will use a variation of the cluster morphism category to compare the two categories of noncrossing partitions. Roughly speaking, there is a projection functor from this new cluster morphism category to the Hubery-Krause category of noncrossing partitions and the fiber category is my category of noncrossing partitions. I will illustrate this in several cases.

Colin Ingalls (University of New Brunswick).
Towards noncommutative resolutions of discriminants.

This is joint work with R. Buchweitz and E. Faber. Let W be subgroup of $\mathrm{GL}(V)$ generated by reflections. Let A be the skew group algebra $W\#k[V]$. Let e be the idempotent of kG corresponding to the trivial representation. We would like to show that A/AeA is a non commutative resolution of the discriminant. Thanks to work of Auslander, Platzeck and Todorov, we can conclude that A/AeA has finite global dimension and is Koszul. We also know that is Cohen-Macaulay and has square rank. We also suspect that it is an endomorphism ring.

Gerard Koffi (University of Iowa).
Deformations of incidence algebras and cohomology.

Incidence algebras were introduced in the 1960's by Gian-Carlo Rota as a way to study combinatorial problems but it became later apparent that such algebras were interesting objects to study on their own; they have applications in geometry and in topology. Examples of incidence algebras include such algebras as the product of copies of a ring R and the upper triangular matrices over R. In this talk, we define deformations of incidence algebras and show that such deformations relate to cohomology. Using distributive modules and quivers, we describe all basic algebras that are deformations of incidence algebras.

Liping Li (UC Riverside).
Koszul property of infinite EI categories.

Some interesting infinite EI categories have been introduced and found to be useful in representation theory, algebraic topology, geometry, and combinatorics. Famous examples include:

- FI, whose objects are finite sets, and morphisms are injections;
- FI_q, whose objects are finite dimensional spaces over a finite fields, and morphisms are linear embeddings;

and many variations. Quite a few important properties of these categories are investigated, such as locally Noetherian property, asymptotic behaviors, and representation stability, etc. In this talk we point out another important property of these categories. That is, their category algebras over a field of characteristic 0 are Koszul. Moreover, we introduce certain combinatorial conditions guaranteeing the Koszul property of category algebras of EI categories of type A_∞. This is a joint work with Wee Liang Gan.

Zongzhu Lin (Kansas State University).
Representations of Rota-Baxter algebras.

Rota-Baxter algebra was introduced by Rota for studying fluctuation theory. It is now used in studying the renormalization in quantum field theory. I will focus on representation theory of Rota-Baxter algebras. The simpliest example is the completion of the ring of meromorphic functions of a curve at a fixed point. We will class all finite dimensional representations and study the orbit structure. These orbits are related to affine Grassmannian. Possible connections with ideal classes will also be discussed.

David Meyer (University of Iowa).
Universal deformation rings for extensions of finite subgroups of $\mathrm{GL}_2(\mathbb{C})$*.*

Let Γ be a finite group, and let V be an absolutely irreducible $\mathbb{F}_p\Gamma$-module. By Mazur, V has a universal deformation ring $R(\Gamma, V)$. This ring is characterized by the property that the isomorphism class of every lift of V over a complete local commutative Noetherian ring R with residue field $\mathbb{F}_p$ arises from a unique local ring homomorphism $\alpha : R(\Gamma, V) \to R$. Let G be a finite subgroup of $\mathrm{GL}_2(\mathbb{C})$. We associate to G a collection of finite groups $\{\Gamma\}$, where each Γ is an extension of G by an elementary abelian p-group N of rank 2, for certain choices of odd primes p. For such a group Γ, a typical absolutely irreducible $\mathbb{F}_p\Gamma$-module V will have universal deformation ring $R(\Gamma, V)$ isomorphic to the p-adic integers $\mathbb{Z}_p$. We discuss those "exceptional" V for which $R(\Gamma, V)$ is not isomorphic to $\mathbb{Z}_p$.

Charles Paquette (University of Connecticut).
Accumulation points of real Schur roots.

Let Q be a connected acyclic quiver with n vertices and let k be an algebraically closed field. A Schur root for Q is a dimension vector of a Schur representation of Q, that is, a representation with trivial endomorphism ring. One way to visualise these roots is to work over the $(n-1)$ real sphere, or just identify vectors up to positive scalars and consider the corresponding rays. When doing this, there is almost no loss of information on Schur roots besides the fact that the ray of an imaginary non-isotropic Schur root contains a family of such roots. We then get well known pictures that are probably familiar to most of you. In such a picture, we can locate the real Schur roots and we can see that they give rise to accumulation points. In this talk, I will try to describe these accumulation points in some nice cases. This is a work in progress.

Daiva Pucinskaite (University of Kiel).
Quivers and relations via the Bruhat order of the symmetric group.

Recall that the Weyl group of the Lie algebra $\mathfrak{sl}(n)$ is isomorphic to the symmetric group $\mathrm{Sym}(n)$. From the Bruhat order on $\mathrm{Sym}(n)$, one would like to read off quiver and relations for the algebra $A(n)$ which describes the principal block of the BGG-category $\mathcal{O}$ of $\mathfrak{sl}(n)$. This presentation deals with the quivers and relations of $A(n)$ and $A(n+1)$ as they are related to the left and right cosets of the subgroup $\mathrm{Sym}(n)$ in $\mathrm{Sym}(n+1)$.

Jennifer Schaefer (Dickinson College).
On the structure of generalized symmetric spaces of $\mathrm{SL}_2(\mathbb{F}_q)$ *and* $\mathrm{GL}_2(\mathbb{F}_q)$*.*

In this talk we will discuss the generalized symmetric spaces for $\mathrm{SL}_2(\mathbb{F}_q)$ and $\mathrm{GL}_2(\mathbb{F}_q)$. Specifically we will characterize the structure of these spaces and prove that when the characteristic of $\mathbb{F}_q$ is not equal to two the extended generalized symmetric space is equal to the generalized symmetric space for $\mathrm{SL}_2(\mathbb{F}_q)$ and nearly equal for $\mathrm{GL}_2(\mathbb{F}_q)$ for all but one involution.

Ian Shipman (University of Michigan).
Some unusual presentations of the algebra $k \times \cdots \times k$.

The algebra $k \times \cdots \times k$ is a very simple algebra which has some surprising properties. I will give two new (as far as I know) presentations of it, one in terms of unary relations alone and the other in terms of noncommutative matrices. These presentations wind up having something to do with branched coverings of algebraic varieties.

Roberto Soto (University of Iowa).
Universal deformation rings and semidihedral 2-groups.

Let k be an algebraically closed field of characteristic 2, let SD be a semidihedral 2-group of order at least 16, and let V be an indecomposable kSD-module. By work of Bleher and Chinburg, V has a universal deformation ring $R(SD,V)$ when the stable endomorphism ring $\underline{\mathrm{End}}_{kSD}(V)$ is isomorphic to k. In this talk, we use work of Carlson and Thévenaz to classify all such kSD-modules, and we discuss how to determine their universal deformation rings.

Hugh Thomas (University of New Brunswick).
Quotients of preprojective algebras and lattice quotients of weak order.

Let B be Dynkin-type preprojective algebra. Mizuno showed that the lattice of torsion classes of B is isomorphic to weak order on the corresponding Weyl group W. We consider the lattice of torsion classes of algebra quotients B/I, and show that they are lattice quotients of weak order on W. I will describe some of the general setting, and then focus on type A, in which we can describe explicitly which lattice quotients arise in this way. We also show that these are exactly the simplicial quotients (i.e., those such that the corresponding coarsening of the Coxeter fan is simplicial). This gives a combinatorial criterion for when a lattice quotient of weak order in the symmetric group is simplicial; no combinatorial criterion was known previously. This work-in-progress is joint with Osamu Iyama, Nathan Reading, and Idun Reiten, and benefitted from initial involvement of David Speyer and Gordana Todorov.

José Vélez-Marulanda (Valdosta State University).
Deformations and derived equivalences over symmetric algebras.

This is joint work with Frauke M. Bleher. Let k be an algebraically closed field of arbitrary characteristic and let Λ be a symmetric k-algebra. In the first part of this talk, we discuss the deformation theory of complexes in the derived category $D^-(\mathrm{PCMod}(\Lambda))$, where $\mathrm{PCMod}(\Lambda)$ denotes the abelian category of pseudocompact Λ-modules. For the second part of the talk, we assume that Γ is another symmetric k-algebra that is derived equivalent to Λ. We discuss the following statement: If${}_\Lambda Q_\Gamma^\bullet$ is the corresponding split-endomorphism two-sided tilting complex (as introduced by Rickard), then ${}_\Lambda Q_\Gamma^\bullet$ preserves the versal deformation rings of complexes in $D^-(\mathrm{PCMod}(\Lambda))$, resp. $D^-(\mathrm{PCMod}(\Gamma))$.

Dan Wackwitz (University of Iowa).
Versal deformation rings of modules over Brauer tree algebras.

Let k be an algebraically closed field of arbitrary characteristic. Suppose A is a Brauer tree algebra over k and V is a finitely generated indecomposable A-module. I am interested in determining the versal deformation ring $R(A, V)$ of V, which is characterized by the property that every lift of V over a complete local commutative Noetherian k-algebra R with residue field k is, up to isomorphism, determined by some (not necessarily unique) local ring homomorphism from $R(A, V)$ to R. In this talk, I will discuss the special case when the Brauer tree of A is a star. In this case, every indecomposable A-module V is uniserial, and I have shown that $R(A, V)$ depends solely on the length of the composition series of V, and can be determined using easily defined matrices.

Peter Webb (University of Minnesota).
Combinatorial restrictions on the AR quiver of a triangulated category.

In a triangulated category with Auslander-Reiten triangles we use additive functions on the quiver (rather than on the tree class) to pin down the possible structures that can occur. We prove that if the quiver has Dynkin tree class then there is only one (shift-)component, thereby generalizing a theorem of Scherotzke, and by analogy with a theorem of Auslander for module categories. We also provide restrictions in the case of extended Dynkin class, and give restrictions on the position of objects in the quiver.

Jerzy Weyman (University of Connecticut).
Local cohomology.

I will discuss the results concerning local cohomology of determinantal varieties, their relation to D-modules and to differential operators as well as Bernstein-Sato polynomials. No previous knowledge of local cohomology will be required.

Gufang Zhao (Institut de Mathématiques de Jussieu).
The elliptic affine Hecke algebra and its representations.

The elliptic affine Hecke algebra is defined by Ginzburg-Kapranov-Vasserot. It is a sheaf of associative algebras on an abelian variety. In this talk, we show that this algebra arises as the convolution algebra of the equivariant elliptic cohomology of the Steinberg variety. As a consequence, the irreducible representations of this algebra are paralell to those of the classical affine Hecke algebra. I will also explain the relation between the elliptic affine Hecke algebra and Cherednik's double affine Hecke algebra. This is a work in progress, joint with Changlong Zhong.