Abstracts

Keynote Lectures

Jerzy Weyman (University of Connecticut).
Noncommutative desingularizations.

In these lectures I will discuss the notion of noncommutative desingularization of an algebraic variety. Usually this means finding the appropriate desingularization of this variety and the tilting object in the category of coherent sheaves over this desingularization. Under certain conditions this gives an equivalence of categories of coherent sheaves over our desingularization and the category of modules over some finite dimensional algebra (the ring of endomorphisms of our tilting module).

Particularly important are the cases when the ring of endomorphisms of the tilting object is a finitely generated maxinal Cohen-Macaulay module over its center. In such cases we talk about a crepant resolution, a noncommutative analogue of a similar notion in algebraic geometry.

In the first lecture I will discuss the definitions and the examples coming from Calabi-Yau varieties, especially rings of invariants of finite subgroups of $\mathrm{SL}(n)$.

In the second lecture I will discuss the work of Buchweitz, Leuschke and Van den Bergh on desingularizations of determinantal varieties as well as some related examples from representation theory. It turns out that in some situations the crepant resolution does not exist so one needs to introduce the concept of categorical crepant resolution.

Conference Talks

Andy Carroll (University of Missouri).
Quiver representations of constant Jordan type.

I will define the notion of a quiver representation of constant Jordan type, a recasting of the notion first studied by Carlson, Friedlander and Pevtsova in the context of finite group schemes. Over appropriately chosen quivers, a coherent sheaf over a projective variety can be associated to any representation of the quiver. In the case that the representation is of constant Jordan type, this sheaf is a vector bundle. I will consider homological interpretations of these modules and pose some open questions. This is joint work with Calin Chindris and Zongzhu Lin.

Giovanni Cerulli Irelli (University of Bonn).
B-orbits of 2-nilpotent elements of classical Lie algebras via symmetric quivers.

We consider either a symplectic or an orthogonal complex Lie group G and the variety X of 2-nilpotent elements in the Lie algebra $\mathfrak{g}$ of G. It is well-known that a Borel subgroup B of G acts with finitely many orbits on X. Following a technique of Boos-Reineke for the case of $G = \mathrm{GL}_n$, we can reduce the study of the B-orbits in X to the study of some symplectic/orthogonal representations of a symmetric quiver with relations. In this talk I will try to explain this reduction and the type of results that one could get using it. This is joint work (still in progress) with G. Carnovale and F. Esposito.

Ted Chinburg (University of Pennsylvania).
Twisting hypercohomology complexes in derived categories.

This talk is about joint work with Ph. Cassou-Noguès, B. Erez and M. J. Taylor. There are many results connecting the Euler characteristics of arithmetically derived sheaves to values of L-series. The hypercohomology complexes of these sheaves are more fundamental than their Euler characteristics. In this talk we relate some new twisting invariants associated to these complexes to values of L-series in the context of finite group actions on varieties over a finite field.

Hailong Dao (University of Kansas).
Some recent results in Cohen-Macaulay representations.

The representation theory of Artin algebras is about studying the category of finite modules M over a k-algebra A, where k is a field. The assumptions naturally force A and M to be finite and free over k. If one replaces k by a regular local ring or a polynomial ring, the subject becomes known as Cohen-Macaulay representation theory. This is rapidly developing and has connections to many areas: algebraic geometry, singularity theory, category theory and noncommutative geometry. I will decribe some recent results, while trying to highlight key similarities and differences with classical representation theory.

Harm Derksen (University of Michigan).
The nullcone for $n \times n$ matrices.

Consider the representation space of the generalized Kronecker quiver with m arrows for the dimension vector (n, n). This space consists of all m tuples of $n \times n$ matrices. The group $\mathrm{SL}_n \times \mathrm{SL}_n$ acts on this space by left and right multiplication. For this action, Hilbert's nullcone consists of all unstable representations of the quiver. This nullcone can be explicitly described geometrically by a result of King on the moduli spaces for quivers. The nullcone is also the zero set of all homogeneous $\mathrm{SL}_n \times \mathrm{SL}_n$-invariants. I will discuss explicit descriptions of invariants that define the nullcone. For $m \leq 2$ this is well known, but I will also discuss the case $n \leq 3$ and a result of two REU students, Sean Douglas and Matt Tanzer, in the case where all matrices are assumed to have rank ≤ 1. These results can be used to obtain degree bounds for generators of the invariant ring, and were motivated by a question of Mulmuley in Geometric Complexity Theory.

Jiarui Fei (UC Riverside).
Tensor Invariants and Kronecker Coefficients.

We are interested in the invariants of $\mathrm{SL}(U) \times \mathrm{SL}(V) \times \mathrm{SL}(W)$ acting on the coordinate ring of $U \times V \times W$ in characteristic zero. We use elementary methods combined with the quiver technology to find explicitly all invariants for $\max\{\dim(U), \dim(V), \dim(W)\} \leq 3$. The dimension of each graded piece is given by a certain Kronecker coefficient. We present an algorithm using quivers to compute general Kronecker coefficients. Finally, we address a conjecture on the degree bounds in the general setting. This conjecture is also related to an important decision problem in the geometric complexity theory about stretched Kronecker coefficients.

Birge Huisgen-Zimmermann (UC Santa Barbara).
Representation-tame algebras need not be homologically tame.

I will start with a brief survey of pre-2000 results on the Finitistic Dimension Conjectures (which date back to 1960), and then discuss progress made post-2000. In particular, I will address:

• Jumps in the functions $n \mapsto findim_n \Lambda$ for $n \in \mathbf{N}$, where Λ is a finite dimensional algebra and $findim_n \Lambda$ is the supremum of the *finite* projective dimensions attained on n-generated left Λ-modules.

• The recent discovery that the first of the conjectures may even fail for representation-tame algebras.

I will conclude with some problems addressing "generic finitistic dimensions".

Miodrag Iovanov (University of Iowa).
Complete path algebras and infinite dimensional representation theory.

While path algebras of finite quivers are a natural framework for the study of representation theory of finite dimenisonal algebras, complete path and monomial algebras are a good framework for the study of a natural infinite dimensional extension of the finite case. The category of locally finite modules over an arbitrary algebra can be thought of as the infinite dimensional generalization of the category of modules over a finite dimensional one. The category of locally finite modules over a complete path or monomial algebras is often the category of locally nilpotent representations of the quiver. We present several results concerning such complete algebras; we give a nil implies nilpotence result as well as a theorem characterizing when a natural torsion splits off for finitely generated modules, which extends classical cases such as modules over the formal power series algebra or PID's. We also investigate an infinite version of the pure semisimplicity conjecture, and show that the natural extension of this question has a negative answer.

Ryan Kinser (Northeastern University/University of Iowa).
Type A quiver loci and Schubert varieties.

We describe a closed immersion from each representation space of a type A quiver with bipartite (i.e., alternating) orientation to a certain opposite Schubert cell of a partial flag variety. This is an analogue of a map introduced by Zelevinsky and later studied by Lakshmibai and Magyar. For type A quivers of arbitrary orientation, we give the same result up to some factors of general linear groups. These identifications allow us to recover results of Bobinski and Zwara; namely we see that orbit closures of type A quivers are normal, Cohen-Macaulay, and have rational singularities. We also see that each representation space of a type A quiver admits a Frobenius splitting for which all of its orbit closures are compatibly Frobenius split.

Ellen Kirkman (Wake Forest University).
Finiteness conditions on the Ext algebra of a monomial algebra.

Let k be a field and let A be a monomial k-algebra, $A = T(V)/I$, where $T(V)$ is a finitely generated tensor k-algebra and I is a set of monomials in $T(V)$. We associate a finite graph $\Gamma(A)$ to A, and use $\Gamma(A)$ to characterize finiteness properties of $\mathrm{Ext}_A(k,k)$, the Yoneda Ext algebra of A, including finite Gelfand-Kirillov dimension, the noetherian property, and finite generation of $\mathrm{Ext}_A(k,k)$. This is joint work with Andrew Conner, James Kuzmanovich, and W. Frank Moore.

Zongzhu Lin (Kansas State University).
Representations of skew polynomial rings and weighted quivers.

Representations of skew polynomial rings are reasonably well understood. In this talk, I describe the cases in positive characteristics when the automorphism is a Frobenius endomorphism and associated geometry for each fixed dimension. The representations of weighted quivers in positive characteristics will also be discussed.

Andras Lorincz (Northeastern University).
The b-functions of quiver semi-invariants.

The main purpose of the talk is to explain how to compute the b-functions of quiver semi-invariants using reflection functors. I will give a quick background on b-functions (also called Bernstein-Sato polynomials) on prehomogeneous vector spaces, and then show by examples how one can compute the b-functions for Dynkin quivers. As a geometric consequence, we have a necessary and sufficient condition for a codimension 1 orbit to have rational singularities.

David Meyer (University of Iowa).
Universal deformation rings in extensions corresponding to faithful representations.

Let Γ be a finite group, and let V be an absolutely irreducible $\mathbb{F}_p\Gamma$-module. By Mazur, V has a universal deformation ring $R(\Gamma, V)$. This ring is characterized by the property that the isomorphism class of every lift of V over a complete local commutative Noetherian ring R with residue field $\mathbb{F}_p$ arises from a unique local ring homomorphism $\alpha : R(\Gamma, V) \to R$. We consider the case when Γ is an extension of a finite group G whose order is relatively prime to p, by an elementary abelian p-group N of rank 2. We further suppose that $\mathbb{F}_p$ is a splitting field for G, and that the action of G on N corresponds to a faithful representation. Such groups G have been classified according to their images in the projective linear group $\mathrm{PGL}_2(\mathbb{F}_p)$. We outline a strategy of how to use this classification to determine to what extent the knowledge of $R(\Gamma, V)$ for all irreducible V can detect the fusion of N in Γ.

Charles Paquette (University of New Brunswick).
Semi-stable equivalence and semi-stable subcategories for Euclidean quivers.

Let Q be a quiver of Euclidean type having n vertices and consider the category $\mathrm{rep}(Q)$ of representations of Q over an algebraically closed field k. A stability condition θ on Q is a homomorphism of abelian groups $\theta : K_0(\mathrm{rep}(Q)) \to \mathbb{Z}$. If $\langle ?, - \rangle$ denotes the Euler homological form for Q, then such a θ can be written as $\theta = \langle d, - \rangle$ for some $d \in \mathbb{Z}^n$. Given a stability condition $\theta = \langle d, - \rangle$, one can consider the category $\mathrm{rep}(Q)_d$ of θ-semi-stable points, which is a nice abelian and extension-closed subcategory of $\mathrm{rep}(Q)$. Given $d_1, d_2 \in \mathbb{Z}^n$, we say that d_1, d_2 are *ss-equivalent* provided $\mathrm{rep}(Q)_{d_1} = \mathrm{rep}(Q)_{d_2}$. We will describe ss-equivalence in a geometric way using an arrangement of convex sets in $\mathbb{Q}^n$ and relate this to canonical decompositions and canonical presentations. This is joint work with C. Ingalls and H. Thomas.

Markus Schmidmeier (Florida Atlantic University).
Varieties of invariant subspaces of linear operators.

Given partitions α, β, γ, the short exact sequences $0 \to N_\alpha \to N_\beta \to N_\gamma \to 0$ of nilpotent linear operators of Jordan types α, β, γ, respectively, define a constructible subset $\mathbb{V}^\beta_{\alpha,\gamma}$ of an affine variety. Geometrically, the $\mathbb{V}^\beta_{\alpha,\gamma}$ are of particular interest as they are naturally occuring varieties which typically consist of several irreducible components. In fact, the components are indexed by the Littlewood-Richardson tableaux Γ of shape (α, β, γ). We describe the closure relation for the components in terms of algebraic and combinatorial partial orders on the set of LR-tableaux. The relations are all equivalent in the case where all parts of α are at most 2. We discuss how the orders differ in general. This is a report about a joint project with Justyna Kosakowska from Torun.

Ian Shipman (University of Michigan).
Some remarks on a conjecture of Orlov.

Orlov conjectured that a variety which admits a full exceptional collection must be rational. I will discuss tilting equivalences for surfaces, and show that Orlov's conjecture for surfaces is related to whether or not a surface with a tilting bundle E is a moduli space of stable representations of $\mathrm{End}(E)$.

Hugh Thomas (University of New Brunswick).
Semistable subcategories for Dynkin and Euclidean quivers.

For Q a Dynkin or Euclidean quiver, I will discuss the categories that can arise as semistable subcategories with respect to a GIT stability condition. The resulting subcategories are exact abelian and extension-closed. In the Dynkin case, all such subcategories arise as semistable subcategories. In the Euclidean case, there are two types of exact abelian and extension-closed subcategories which appear: those which admit a relative projective generator, and the regular component of such a subcategory if the subcategory is of infinite type. I will discuss how to classify these subcategories by certain elements of the Weyl group corresponding to Q.

Gordana Todorov (Northeastern University).
Lattice structure of torsion classes for hereditary algebras.

This is joint work with Osamu Iyama, Idun Reiten and Hugh Thomas. We study categories of modules, $\operatorname{mod} A$, over hereditary artin algebras, and torsion classes in $\operatorname{mod} A$. For these algebras, torsion classes always form a lattice. We concentrate on functorially finite torsion classes and show that functorially finite torsion classes form a lattice precisely when the only rigid modules are preprojective and preinjective modules. It is known that functorially finite torsion classes are closely related to tilting modules and support tilting modules hence our theorem defines poset structure on the support tilting modules, precisely when the quiver is Dynkin or has exactly 2 vertices.

José Vélez-Marulanda (Valdosta State University).
Deformations and derived equivalences over Frobenius algebras.

This is joint work with Frauke M. Bleher. Let k be an algebraically closed field of arbitrary characteristic and let Λ be a Frobenius k-algebra. In this talk, we discuss the deformation theory of complexes in the derived category $D^-(\mathrm{PCMod}(\Lambda))$, where $\mathrm{PCMod}(\Lambda)$ denotes the abelian category of pseudocompact Λ-modules. In case Λ and Γ are symmetric k-algebras that are derived equivalent to each other we also discuss the following statement: If${}_\Lambda Q^\bullet_\Gamma$ is the corresponding split-endomorphism two-sided tilting complex (as introduced by Rickard), then ${}_\Lambda Q^\bullet_\Gamma$ preserves the versal deformation rings of complexes in $D^-(\mathrm{PCMod}(\Lambda))$, resp. $D^-(\mathrm{PCMod}(\Gamma))$.

Gufang Zhao (Northeastern University).
Stability conditions for noncommutative symplectic resolutions.

This is a preliminary report. A localization theorem for the cyclotomic rational Cherednik algebra $H_c = H_c((\mathbb{Z}/l)^n \rtimes \mathfrak{S}_n)$ over a field of positive characteristic has been proved by Bezrukavnikov, Finkelberg and Ginzburg. Localizations with different parameters give different t-structures on the derived category of coherent sheaves on the Hilbert scheme of points on a surface. In this talk I will concentrate on the comparison between different t-structures coming from different localizations. When $n = 2$, I will show an explicit construction of tilting bundles that generates these t-structures. These t-structures are controlled by a real variation of stability conditions, a notion related to Bridgeland stability conditions. I will also talk about its relation to the topology of Hilbert schemes and irreducible representations of H_c.