# Abstracts

## Keynote Lectures

### Birge Huisgen-Zimmermann (UC Santa Barbara).

*Fine and coarse moduli spaces in the representation theory of wild algebras.*

We will begin by introducing the concepts of fine/coarse moduli spaces, originally devised with an eye to classification problems within algebraic geometry. After a brief history and some examples recruited from time-honored settings, we will discuss two fundamentally different lines of approach in applying these notions to classification problems in the representation theory of finite dimensional algebras. One is based on the classical affine parametrizing varieties of modules. It singles out accessible subsets in terms of stability, a method that was developed by Mumford with the goal of classifying vector bundles over certain projective varieties. It was adjusted to the scenario of quiver representations by King, and exploited by numerous researchers in the sequel. We will explain the theoretical underpinnings and provide some concrete examples. The alternate geometric access to representations is by way of projective parametrizing varieties, located inside suitable subspace Grassmannians; these alternate module varieties encode minimal projective presentations. Since several surveys on the former approach are available, we will place stronger emphasis on the latter. In particular, we will discuss and illustrate a fairly extensive sampler of results obtained by way of Grassmann varieties and point to open problems. Moreover, we will explain how to move back and forth between the two geometric settings to integrate information. The juxtaposition of the two methods will highlight their differences. These differences not only concern modes of argumentation to arrive at the same ends, but – in the first place – affect expectations and attainable goals along the two lines.

## Conference Talks

**Jon Carlson (University of Georgia).**
*Thick subcategories of the bounded derived category.*

This is joint work with Srikanth Iyengar. It is all about using methods from commutative algebra to study group representations. A new proof of the classification for tensor ideal thick subcategories of the bounded derived category, and the stable category, of modular representations of a finite group is obtained. The arguments apply more generally to yield a classification of thick subcategories of the bounded derived category of an artinian complete intersection ring. One of the salient features of this work is that it takes no recourse to infinite constructions, unlike the previous proofs of these results.

**Andy Carroll (University of Missouri).**
*Rational invariants for gentle algebras.*

We will discuss new results on fields of rational invariants and moduli spaces for irreducible components of represntation spaces over gentle algebras. These results confirm for gentle algebras conjectures by both Chindris and Weyman concerning the relationship between tameness of an algebra and the structure of fields of rational invariants and moduli spaces. This is joint work with C. Chindris.

**Ted Chinburg (University of Pennsylvania).**
*Orbit closures and rational surfaces.*

This talk is about joint work with Frauke Bleher and Birge Huisgen-Zimmermann on bounding the geometry of the orbit closures arising from submodules $C$ of given dimension in a projective module $P$ for a finite dimensional algebra $\Lambda$ over an algebraically closed field $k$. We consider the case in which the orbit of $\mathrm{Aut}_\Lambda(P)$ acting on $C$ in the appropriate Grassmannian is an affine plane $\mathbb{A}^2_k$. We first show that the orbit closure is dominated by a “bad blow-up” $B$ of $\mathbb{P}^1_k \times \mathbb{P}^1_k$. We then show that $B$ can be dominated by a bounded number of “good blow-ups” of one of a bounded number of relatively minimal smooth rational projective surfaces $X$. The bounds depend only on $k$ and $\dim_k(P)$. We do not know whether all such $X$ can arise; we will discuss three such $X$ which do occur.

**Harm Derksen (University of Michigan).**
*Mutations of quivers with potentials.*

Quivers with potentials have been used by Jerzy Weyman, Andrei Zelevinsky and myself to prove conjectures about cluster algebras with coefficients. In this talk, I will describe the generalization of the Bernstein-Gelfand-Ponomarev reflection functors to mutations of representations of quivers with potentials, and the interesting combinatorial properties of these mutations.

**Jiarui Fei (UC Riverside).**
*Wall-crossings of moduli of quiver representations.*

In this talk, I will explain how a GIT quotient space of quiver representations changes when its stability condition crosses special walls given by real Schur roots of the quiver. The essential case is proven to be the usual blow-up along subvarieties. The blow-up actually happens at a GIT quotient of representations of a new quiver with one vertex less. The new quiver with dimension vector and stability condition, and the blow-up loci can be explicitly described. Moreover, a wall crossing formula for the induced ample divisor is derived. I will illustrate this theory by many examples.

**Frederico Galetto (Northeastern University).**
*Representations with finitely many orbits and free resolutions.*

The representations of reductive groups with finitely many orbits are parametrized by graded simple Lie algebras. For the exceptional Lie algebras, Kraśkiewicz and Weyman exhibit the expected minimal free resolutions for the coordinate ring of the normalization of the orbit closures. I will present an interactive method to verify their conjectures using Macaulay2. Given time, I will also mention how to use these free resolutions to study other related modules supported on the orbit closures.

**Tom Howard (Syracuse University).**
*Complexity classes and stable derived equivalences of finite dimensional algebras.*

I will construct some stable derived equivalences of finite dimensional algebras and discuss implications for a conjecture that complexity is a stable derived invariant.

**Miodrag Iovanov (University of Iowa).**
*Frobenius-Artin algebras and infinite linear codes.*

We show that Frobenius rings that are at the same time Artin algebras have characterizations that extend and unify well known results of Nakayama for algebras over fields, and some recent results for finite Frobenius rings [T. Honold, Arch. Math (Basel) 76, no. 6 (2001)], and which entitles one to call such algebras Frobenius-Artin algebras. On the other hand, finite Frobenius rings have raised interest due to connections with coding theory [J.A. Wood, Proc. AMS 136, no. 2 (2008)]. It has been recently shown that they are characterized as rings for which linear codes have the extension property [J.A. Wood, Amer. J. Math 121, no. 3 (1999)]. We generalize this to arbitrary rings, and show that in the infinite case, the categorical properties of Frobenius rings are the captured by this extension property. Namely, we show that a ring has the extension property for linear codes if and only if it is the product of a finite Frobenius ring and a quasi-Frobenius ring with no finite representations (modules). We give two proofs of this, one that uses measure theory and compact groups, and another combinatorial one.

**Ryan Kinser (Northeastern University).**
*Dense orbit algebras.*

I will discuss some examples of “DO algebras” which were introduced in my recent joint work with Calin Chindris and Jerzy Weyman. In particular, I will give an example of an algebra of wild representation type which admits classification of its generic representations by geometric methods, and give some idea how to achieve such a classification.

**Ellen Kirkman (Wake Forest University).**
*Invariants of $(-1)$-skew polynomial rings under permutation representations.*

This is joint work with J. Kuzmanovich and J. Zhang. Let $k$ be a field of characteristic zero, let $A = k_{-1}[x_1, ..., x_n]$ be the skew polynomial ring with $x_j x_i = -x_i x_j$ for $i \neq j$, and let $G$ be any group of graded automorphisms of $A$ induced by a set of permutations of $\{x_i\}$. We show that the invariant subrings $A^G$ are AS Gorenstein, and, when $G$ is the full symmetric group $S_n$ or the alternating group $A_n$, the invariant subrings $A^G$ are factors of AS-regular algebras by regular homogeneous normalizing elements. For arbitrary $G \subseteq S_n$, we obtain upper bounds on the degrees of algebra generators of $A^G$ that parallel those of Broer and Göbel when $A = k[x_1, ..., x_n]$.

**Zongzhu Lin (Kansas State University).**
*Counting representations and Kac conjecture.*

Given a finite quiver $Q$ without loops, there is an associated Kac-Moody Lie algebra $\mathfrak{g}$. Kac proved that indecomposable finite dimensional representations of $Q$ over a field have dimension vectors exactly being positive roots of $\mathfrak{g}$. Kac conjectured that the number of absolutely indecomposable representations with a fixed dimension factor $\alpha$ over a finite field with $q$ elements is a polynomial of $q$ with non-negative integer coefficients and the constant term is exactly the dimension of the root space of $\mathfrak{g}$ of the positive root $\alpha$. This conjecture has been proved recently by Hausel et al. The main idea of the proof involves using arithmetic Fourier transforms to count the number of rational points of algebraic varieties over finite fields and to relate them to Betti numbers of Nakajima quiver varieties. In this talk, I will outline the main ideas and possible equivariant variations.

**David Meyer (University of Iowa).**
*Universal deformation rings and fusion.*

Let $\Gamma$ be a finite group, and let $V$ be an absolutely irreducible $\mathbb{F}_p\Gamma$-module. By work of Mazur, $V$ has a universal deformation ring $R(\Gamma, V)$ whose ring structure is closely related to the second cohomology group $\mathrm{H}^2(\Gamma, \mathrm{Hom}_{\mathbb{F}_p}(V, V))$. In this talk, we consider the case when $\Gamma$ is an extension of a $p'$-group $G$ by an elementary abelian $p$-group $N$ and compare the dimension of $\mathrm{H}^2(\Gamma, \mathrm{Hom}_{\mathbb{F}_p}(V, V))$ to the fusion of $N$ in $\Gamma$.

**Jennifer Schaefer (Dickinson College).**
*Universal deformation rings and tame blocks with two simple modules.*

This is joint work with Frauke Bleher and Giovanna Llosent. Let $k$ be an algebraically closed field of characteristic 2. Suppose $G$ is a finite group and $B$ is a block of $kG$ with a semi-dihedral or a generalized quaternion defect group and precisely two isomorphism classes of simple $B$-modules. In this talk, we establish the list of all finitely generated $kG$-modules $V$ which belong to $B$ and whose endomorphism ring is isomorphic to $k$ using Erdmann's description of the quiver and relations of the basic algebra of $B$. The goal of our work is to use this result to determine the universal deformation ring $R(G, V)$ for every finitely generated $kG$-module $V$ which satisfies these properties.

**Markus Schmidmeier (Florida Atlantic University).**
*Arc diagram varieties.*

Let $k$ be an algebraically closed field and let $\alpha$, $\beta$, $\gamma$ be partitions. An algebraic group acts on the constructible set of short exact sequences

$$0 \longrightarrow N_\alpha \longrightarrow N_\beta \longrightarrow N_\gamma \longrightarrow 0$$

of nilpotent $k$-linear operators of Jordan types $\alpha$, $\beta$, and $\gamma$, respectively, such that the orbits correspond to the equivalence classes. In the case where all parts of $\alpha$ are at most 2, the orbits are in one-to-one correspondence with certain arc diagrams. Geometric properties of this stratification, in particular the dimensions of the orbits, the degeneration relation, and the number of strata of maximal and minimal dimension, are controlled by the combinatorics of arc diagrams. In this talk we consider a problem posed by Birge Huisgen-Zimmermann on whether all saturated chains connecting two given strata have the same length. While this property does not hold in general, we can use for certain partition triples $(\alpha, \beta, \gamma)$ the extended bubble sort algorithm to refine chains of strata such that subsequent dimension differences are equal to one. This is a report on a joint project with Justyna Kosakowska from Torun.

**Ian Shipman (University of Michigan).**
*Remarks on degenerations and rational self maps of Grassmannians.*

I will report on some preliminary joint work with Ryan Kinser. Let $A$ be a finite dimensional associative algebra and $P$ a finitely generated projective $A$-module. Put $d = \dim(\mathrm{top}(P))$. Then every element of $\mathrm{Aut}(P)(\mathbb{C}[t, t^{-1}])$ determines a rational self map of $\mathrm{Gr}(P, d)$, which takes a quotient to a degeneration of this quotient. Elements of $\mathrm{Aut}(P)(\mathbb{C}[t, t^{-1}])$ can be decomposed into simple factors. For each type of simple factor, I will describe the correspondence resolving its rational map in representation theoretic terms.

**José Vélez-Marulanda (Valdosta State University).**
*Universal deformation rings of modules over a certain symmetric tame algebra.*

Let $K$ be an algebraically closed field, let $\Lambda$ be a finite dimensional $K$-algebra and let $V$ be a $\Lambda$-module with stable endomorphism ring isomorphic to $K$. If $\Lambda$ is self-injective then $V$ has a universal deformation ring $R(\Lambda, V)$, which is a complete local commutative Noetherian $K$-algebra with residue field $K$. Moreover, if $\Lambda$ is also a Frobenius $K$-algebra then $R(\Lambda, V)$ is stable under syzygies. We use these facts to determine the universal deformation rings of $\Lambda_{(r_0,r_1,r_2,k)}$-modules whose stable endomorphism ring is isomorphic to $K$, where $\Lambda_{(r_0,r_1,r_2,k)}$ is a symmetric special biserial $K$-algebra that has quiver with relations depending on the four parameters $r_0, r_1, r_2 \geq 3$ and $k \geq 2$. Our goal is to explain how universal deformation rings change when inflating modules from $\Lambda_{(r_0,r_1,r_2,k)}$ to $\Lambda_{(r'_0,r'_1,r'_2,k')}$, where $\Lambda_{(r'_0,r'_1,r'_2,k')}$ surjects onto $\Lambda_{(r_0,r_1,r_2,k)}$ when $r'_0 \geq r_0$, $r'_1 \geq r_1$, $r'_2 \geq r_2$, $k' \geq k$.

**Chelsea Walton (M.I.T.).**
*Quantum binary polyhedral groups and their actions on quantum planes.*

We quantize results in classical invariant theory, namely results on the invariants of finite group actions on the commutative polynomial ring $\mathbb{C}[u, v]$. First, we classify quantum analogues of finite subgroups of $\mathrm{SL}_2(k)$, and second, we study their actions on Artin-Schelter (AS) regular algebras of global dimension two. Note that AS regular algebras play the role of noncommutative polynomial rings in noncommutative projective algebraic geometry. Properties of the resulting invariant rings will also be given.

**Gufang Zhao (Northeastern University).**
*Noncommutative desingularization of orbit closures for some* $\mathrm{GL}_n$ *representations.*

A noncommutative desingularization of determinantal varieties defined by maximal minors of generic matrices has been studied by Buchweitz, Leuschke, and van den Bergh. This talk will be on a generalization of their results to the orbit closures of other $\mathrm{GL}_n$-representations. I will describe a method to calculate the quiver with relations for any noncommutative desingularizations coming from exceptional collections over partial flag varieties. A notion of equivariant quivers will be introduced, as a language to describe these noncommutative desingularizations. Explicit expression of noncommutative desingularizations can be given for generic determinantal varieties, determinantal varieties defined by minors of generic symmetric matrices, and pfaffian varieties defined by pfaffians of generic skew-symmetric matrices. For maximal minors of square matrices and symmetric matrices, this gives a noncommutative crepant resolution. This talk is based on my joint work with Jerzy Weyman.