# BIRS WORKSHOP

## "NEW DIRECTIONS IN IWASAWA THEORY" JUNE 26 – JULY 1, 2016

# Schedule

# SUNDAY, JUNE 26

16:00	Check-in begins
	(Front Desk - Professional Development Center - open 24 hours)
17:30 - 19:30	Buffet Dinner, Sally Borden Building
19:30	Informal gathering (Corbett Hall, 2nd floor lounge)

# Monday, June 27

7:00 - 8:45	Breakfast
8:45 - 9:00	Introduction and welcome by BIRS station manager
9:00 - 9:50	Chinburg: Iwasawa theory in higher codimension.
10:00 - 10:30	Coffee break
10:30 - 11:20	Popescu: Towards module structure in classical Iwasawa theory.
11:30 - 13:00	Lunch
13:00 - 13:50	Guided Tour of The Banff Centre
	(meet in the 2nd floor lounge, Corbett Hall)
13:50 - 14:00	Group Photo (meet in foyer of TCPL)
14:00 - 14:50	Dasgupta: On the Gross-Stark Conjecture.
15:00 - 15:30	Coffee break
15:30 - 16:20	Castella: A-adic Gross-Zagier formula for elliptic curves at supersingular primes.
16:40 - 17:30	Palvannan: On Selmer groups and factoring p-adic L-functions.
17:45 - 19:30	Dinner

# TUESDAY, JUNE 28

7:00 - 9:00	Breakfast
9:00 - 9:50	Hida: Ring theoretic properties of Hecke algebras and cyclicity in Iwasawa theory.
10:00 - 10:30	Coffee break
10:30 - 11:20	Kurihara: Iwasawa theory and Rubin-Stark elements.
11:30 - 13:30	Lunch
14:00 - 14:50	Witte: On zeta-isomorphisms and main conjectures.
15:00 - 15:30	Coffee break
15:30 - 16:20	Büyükboduk: On the anticyclotomic main conjectures for modular forms.
16:40 - 17:30	Sprung: The main conjecture for elliptic curves at non-ordinary primes.
17:45 - 19:30	Dinner

## Wednesday, June 29

7:00 - 9:00 9:00 - 9:50 10:00 - 10:20 10:20 - 11:10 11:30 - 12:20 12:30 - 13:30	Breakfast Rubin: Heuristics for the growth of Mordell-Weil ranks in big extensions. Coffee break Venjakob: Wach modules, regulator maps and $\varepsilon$ -isomorphisms in families. Fouquet: Congruences between motives and values of L-functions. Lunch Free Afternoon Dinner		
Thursday, June 30			
7:00 - 9:00 9:00 - 9:50 10:00 - 10:30 10:30 - 11:20 11:30 - 13:30 14:00 - 14:50 15:00 - 15:30 15:30 - 16:20 16:40 - 17:30 17:45 - 19:30	Breakfast Schneider: Rigid character groups, Lubin-Tate theory, and $(\varphi, \Gamma)$ -modules. Coffee break Çiperiani: Local points of supersingular elliptic curves on $\mathbb{Z}_p$ -extensions. Lunch Hsieh: Hida families and triple product p-adic L-functions. Coffee break Burungale: On $\mathfrak{p}$ -anticyclotomic Iwasawa theory. Wake: Ordinary pseudorepresentations, modular forms & Iwasawa theory. Dinner		

## FRIDAY, JULY 1

7:00 - 9:00	Breakfast
9:00 - 10:00	Discussions
10:00 - 10:30	Coffee break
10:30 - 11:30	Discussions
11:30 - 12:00	Checkout by Noon (Front Desk - Professional Development Centre)
12:00 - 13:30	Lunch

## **Remarks**:

- All lectures will be held in the TransCanada Pipelines Pavilion (TCPL).
- All meals will be served in the Sally Borden Building: Breakfast: 7:00 - 9:30 (Monday - Friday), Lunch: 11:30 - 13:30 (Monday - Friday), Dinner: 17:30 - 19:30 (Sunday - Thursday).
- The coffee breaks are in the foyer of TCPL.
- All participants are required to check out by 12 noon on Friday, August 21.

#### Abstracts

## Ashay Burungale (University of Arizona).

On p-anticyclotomic Iwasawa theory.

Let F be a totally real field. Let p be an odd prime unramified in F and  $\mathfrak{p}$  a prime above p. Let K/F be a p-ordinary CM quadratic extension and  $K_{\mathfrak{p}}^-$  the maximal p-anticyclotomic extension of K unramified outside  $\mathfrak{p}$ . We discuss results on the  $\mu$ -invariant of certain p-adic L-functions over K along the  $\mathfrak{p}$ -anticyclotomic tower. We also describe relevant questions regarding the  $\mathfrak{p}$ -anticyclotomic Selmer groups (joint with H. Hida).

### Kazım Büyükboduk (Koc University of Istanbul).

On the anticyclotomic main conjectures for modular forms.

I will report on recent joint work with Antonio Lei on the anticyclotomic Iwasawa theory of the base change of an elliptic modular form to an imaginary quadratic field K in which the prime p splits. We treat both the definite and indefinite cases in both p-ordinary and non-p-ordinary situations. One of our main results is an equality (up to powers of p) that is predicted by the main conjectures in the definite p-ordinary set up and a  $\Lambda$ -adic Birch and Swinnerton-Dyer formula in the indefinite case.

## Francesc Castella (UC Los Angeles).

 $\Lambda$ -adic Gross-Zagier formula for elliptic curves at supersingular primes.

In 2013, Kobayashi proved an analogue of Perrin-Riou's *p*-adic Gross-Zagier formula for elliptic curves at supersingular primes. In this talk, we will explain an extension of Kobayashi's result to the  $\Lambda$ -adic setting. The main formula is in terms of plus/minus Heegner points up the anticyclotomic tower, and its proof, rather than on calculations inspired by the original work of Gross-Zagier, is via Iwasawa theory, based on the connection between Heegner points, Beilinson-Flach elements, and their explicit reciprocity laws. This is joint work with Xin Wan.

#### Ted Chinburg (University of Pennsylvania).

Iwasawa theory in higher codimension.

This talk will be a survey of recent work on higher codimension Iwasawa theory (joint with F. Bleher, R. Greenberg, M. Kakde, G. Pappas, R. Sharifi and M. Taylor). This has to do with relating p-adic L-functions to the behavior of Iwasawa modules which are supported in codimension larger than one as modules for an Iwasawa algebra. One idea I will discuss is that the natural analytic invariants arising from Katz p-adic L-functions pertain to the derived top exterior powers of Iwasawa modules. For first Chern classes, passing to the derived top exterior power makes no difference, but for higher Chern classes it does. This is analogous to the fact that Stark's conjectures pertain to regulators rather than to the individual entries of matrices whose determinants are regulators.

#### Mirela Çiperiani (UT Austin).

Local points of supersingular elliptic curves on  $\mathbb{Z}_p$ -extensions.

Work of Kobayashi and Iovita-Pollack describes how local points of supersingular elliptic curves on ramified  $\mathbb{Z}_p$ -extensions of  $\mathbb{Q}_p$  split into two strands of even and odd points. We will discuss a generalization of this result to  $\mathbb{Z}_p$ -extensions that are localizations of anticyclotomic  $\mathbb{Z}_p$ -extensions over which the elliptic curve has non-trivial CM points.

#### Samit Dasgupta (UC Santa Cruz).

On the Gross-Stark Conjecture.

In 1980, Gross conjectured a formula for the expected leading term at s = 0 of the Deligne–Ribet *p*-adic *L*-function associated to a totally even character  $\psi$  of a totally real field *F*. The conjecture states that after scaling by  $L(\psi\omega^{-1}, 0)$ , this value is equal to a *p*-adic regulator of units in the abelian extension of *F* cut out by  $\psi\omega^{-1}$ . In this talk we describe a proof of Gross's conjecture. This is joint work with Mahesh Kakde and Kevin Ventullo.

#### Olivier Fouquet (Université Paris-Sud).

Congruences between motives and congruences between values of L-functions.

If two motives are congruent, is it the case that the special values of their respective L-functions are congruent, or – more precisely – can the formula predicting special values of motivic L-functions be interpolated in p-adic families of motives? I will explain how the formalism of the Weight-Monodromy filtration for p-adic families of Galois representations sheds light on this question (and suggests a perhaps surprising answer).

## Haruzo Hida (UC Los Angeles).

Ring theoretic properties of Hecke algebras and cyclicity in Iwasawa theory.

We can formulate certain Gorenstein property of subrings of the universal deformation ring (i.e., the corresponding Hecke algebra) as a condition almost equivalent to the cyclicity of the Iwasawa module over  $\mathbb{Z}_p$ -extensions of an imaginary quadratic field if the starting residual representation is induced from the imaginary quadratic field. I will discuss this fact in some details.

#### Ming-Lun Hsieh (National Taiwan University).

Hida families and triple product p-adic L-functions.

In this talk, we will present a construction of the three-variable p-adic L-function attached to the triple product of three Hida families. This p-adic L-function is a three-variable power series with p-integral coefficients interpolating central L-values of triple product L-functions in the balanced case. We will give the explicit interpolation formula at all critical specialisations and discuss some problems on this p-adic L-function.

## Masato Kurihara (Keio University).

Iwasawa theory and Rubin-Stark elements.

We will discuss Rubin-Stark elements and zeta elements Iwasawa theoretically, and discuss equivariant main conjectures and their consequences.

#### Bharathwaj Palvannan (University of Washington).

On Selmer groups and factoring p-adic L-functions.

Haruzo Hida has constructed a 3-variable Rankin Selberg p-adic L-function. Two of its variables are "weight" variables and one of its variables is the "cyclotomic" variable. Samit Dasgupta has factored a certain restriction of this 3-variable p-adic L-function (when the two weight variables are set equal to each other) into a product of a 2-variable p-adic L-function (related to the adjoint representation of a Hida family) and the Kubota-Leopoldt p-adic L-function. We prove the corresponding result involving Selmer groups that is predicted by the main conjectures. A key technical input is studying the (height one) specialization of Selmer groups.

## Cristian Popescu (UC San Diego).

Towards module structure in classical Iwasawa theory.

I will discuss aspects of my recent joint work with Corey Stone on higher Fitting ideals of various Iwasawa modules. In particular, I will discuss a conjecture of Kurihara in this direction.

## Karl Rubin (UC Irvine).

Heuristics for the growth of Mordell-Weil ranks in big extensions of number fields.

I will discuss some heuristics for modular symbols, and consequences of those heuristics for Mordell-Weil ranks. For example, these heuristics predict that every elliptic curve over  $\mathbb{Q}$  has finite Mordell-Weil rank over the  $\mathbb{Z}$ -extension of  $\mathbb{Q}$ . This is joint work with Barry Mazur.

#### Peter Schneider (Universität Münster).

Rigid character groups, Lubin-Tate theory, and  $(\varphi, \Gamma)$ -modules.

The talk will describe joint work with L. Berger and B. Xie in which we build, for a finite extension L of  $\mathbb{Q}_p$ , a new theory of  $(\varphi, \Gamma)$ -modules whose coefficient ring is the ring of holomorphic functions on the rigid character variety of the additive group  $o_L$ , resp. a "Robba" version of it.

#### Florian Sprung (Princeton University).

The main conjecture for elliptic curves at non-ordinary primes.

We explain the proof of the main conjecture for elliptic curves at non-ordinary primes. This generalizes work of Wan, who worked under the assumption that  $a_p = 0$ .

#### Otmar Venjakob (Universität Heidelberg).

Wach modules, regulator maps and  $\varepsilon$ -isomorphisms in families.

In this talk on joint work with Rebecca Bellovin we discuss the "local  $\varepsilon$ -isomorphism" conjecture of Fukaya and Kato for (crystalline) families of  $G_{\mathbb{Q}_p}$ -representations. This can be regarded as a local analogue of the global Iwasawa main conjecture for families, extending earlier work of Kato for rank one modules, of Benois and Berger for crystalline representations with respect to the cyclotomic extension, as well as of Loeffler, Venjakob and Zerbes for crystalline representations with respect to abelian *p*-adic Lie extensions of  $\mathbb{Q}_p$ . Nakamura has shown Kato's conjecture for ( $\varphi, \Gamma$ )-modules over the Robba ring, which means in particular only after inverting *p*, for rank one and trianguline families. The main ingredient of (the integrality part of) the proof consists of the construction of families of Wach modules generalizing work of Wach and Berger and following Kisin's approach via a corresponding moduli space.

## Preston Wake (UC Los Angeles).

Ordinary pseudorepresentations, modular forms and Iwasawa theory.

Pseudorepresentations appear naturally when we talk about modular forms that are congruent to Eisenstein series. I'll talk about the difficulties that arise when defining "ordinary pseudorepresentation", and how to resolve these difficulties. I'll explain how the deformation theory of pseudorepresentations is related to cyclotomic Iwasawa theory and the geometry of the ordinary eigencurve. This is joint work with Carl Wang Erickson.

## Malte Witte (Universität Heidelberg).

On zeta-isomorphisms and main conjectures.

The zeta-isomorphism conjecture of Fukaya and Kato is a generalisation of the equivariant Tamagawa number conjecture. I will briefly explain the general setup of the conjecture. I then turn to the noncommutative main conjecture for totally real fields and discuss a unicity result for the noncommutative zeta functions constructed by Kakde. Finally, I explain how this unicity result can be used to construct zeta-isomorphisms in the sense of Fukaya and Kato.