

Numerical Example of Spline Interpolation

Let $f(x) = e^x$, $0 \leq x \leq 1$. For $n > 0$, let $h = 1/n$ and $x_j = jh$, $j = 0, 1, \dots, n$.

Case 1. Natural cubic spline interpolation

Case 2. Clamped cubic spline interpolation

Case 3. Cubic spline interpolation with true second derivatives

Case 4. Cubic spline interpolation with the “not-a-knot” condition

Table 1: Maximum Error on $[0, 1]$

<i>Case</i>	<i>n</i> = 10	<i>Ratio</i>	<i>n</i> = 20	<i>Ratio</i>	<i>n</i> = 40
1	1.24E-3	3.99	3.11E-4	4.00	7.77E-5
2	6.96E-7	15.9	4.39E-8	16.0	2.75E-9
3	1.71E-6	15.8	1.08E-7	15.9	6.78E-9
4	6.40E-6	15.2	4.21E-7	15.6	2.70E-8

The results on smaller intervals lead to somewhat improved results for case (1).

Table 2: Maximum Errors for Case (1)

<i>Interval</i>	<i>n</i> = 10	<i>Ratio</i>	<i>n</i> = 20	<i>Ratio</i>	<i>n</i> = 40
[0.1, 0.9]	3.22E-4	14.4	2.23E-5	56.0	3.98E-7
[0.2, 0.8]	8.86E-5	56.4	1.57E-6	575	2.73E-9

These results show the “not-a-knot” condition is preferable if you do not know derivatives of f , since such are needed for cases 2 and 3.