THE NUMERICAL SOLUTION OF INTEGRAL EQUATIONS OF THE SECOND KIND [1]

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PREFACE

In this book, numerical methods are presented and analyzed for the solution of integral equations of the second kind, especially Fredholm integral equations. Major additions have been made to this subject in the 20 years since the publication of my survey [39], and I present here an up-to-date account of the subject. In addition, I am interested in methods which are suitable for the solution of boundary integral equation reformulations of Laplace's equation, and three chapters are devoted to the numerical solution of such boundary integral equations. Boundary integral equations of the first kind which have a numerical theory closely related to that for boundary integral equations of the second kind are also discussed.

This book is directed towards several audiences. It is first directed to numerical analysts working on the numerical solution of integral equations. Second, it is directed towards applied mathematicians, including both those interested directly in integral equations and those interested in solving elliptic boundary value problems by use of boundary integral equation reformulations. Finally, it is directed towards that very large group of engineers needing to solve problems involving integral equations. In all of these cases, I hope the book is also readable and useful to well-prepared graduate students, as I had them in mind when writing the book.

During the period of 1960-1990, there has been much work on developing and analyzing numerical methods for solving linear Fredholm integral equations of the second kind, with the integral operator being compact on a suitable space of functions. I believe this work is nearing a stage in which there will be few major additions to the theory, especially as regards equations for functions of a single variable. In Chapters 2 through 6, the main aspects of the theory of numerical methods for such integral equations is presented, including recent work on solving integral equations on surfaces in $\mathbf{R}^3$. Chapters 7 through 9 contain a presentation of numerical methods for solving some boundary integral equation reformulations of Laplace's equation, for problems in both two and three dimensions. By restricting the presentation to Laplace's equation, a simpler development can be given than is needed when dealing with the large variety of boundary integral equations which have been studied during the past twenty years. For a more complete development of the numerical solution of all forms of boundary integral equations for planar problems, see Prößdorf and Silbermann [438].

In Chapter 1, a brief introduction/review is given of the classical theory of Fredholm integral equations of the second kind in which the integral operator is compact. In presenting the theory of this and the following chapters,a functional analysis framework is used, which it is generally set in the space of continuous functions $C(D)$ or the space of square integrable functions $L^2(D)$. Much recent work has been in the framework of Sobolev spaces $H^r(D)$, which is used in portions of Chapter 7; but I believe the simpler framework given here is accessible to a wider audience in the applications community. Therefore I have chosen this simpler framework in preference to regarding boundary integral equations as pseudodifferential operator equations on Sobolev spaces. The reader still will need to have some knowledge of functional analysis, although it is not a great deal; and a summary of some of the needed results from functional analysis is presented in the appendix.

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