MODELLING POPULATION GROWTH

$P(t)$ = population of USA in year t, in thousands.

t	$P(t)$	$\frac{P(t+10)-P(t)}{10} \div P(t)$
1790	3929	.035
1800	5308	.036
1810	7240	.033
1820	9638	.033
1830	12866	.033
1840	17069	.036
1850	23192	.036
1860	31443	.027
1870	39818	

The average of the values in the last column is 0.0346. Using

$$P'(t) \approx \frac{P(t+10)-P(t)}{10}$$

we arrive at the differential equation

$$N'(t) = .0346N(t), \quad N(1790) = 3929$$

We use a new variable $N(t)$ to distinguish the value obtained by our model from the true value $P(t)$. How well does the model predict the actual population?

The solution of

$$N'(t) = .0346N(t), \quad N(1790) = 3929$$

is

$$N(t) = 3929e^{.0346(t-1790)}$$

As another model, consider

$$M'(t) = .02975M(t), \quad M(1790) = 3929$$

Its solution is

$$M(t) = 3929e^{.02975(t-1790)}$$

We use this model as the growth rate of 0.02975 turns out to be more accurate.

The models lead to the following table of values.

t	$N(t)$	$M(t)$	$P(t)$
1790	3929	3929	3929
1800	5553	5290	5308
1810	7849	7123	7240
1820	11094	9592	9638
1830	15680	12915	12866
1840	22162	17390	17069
1850	31324	23415	23192
1860	44273	31528	31443
1870	62576	42452	39818