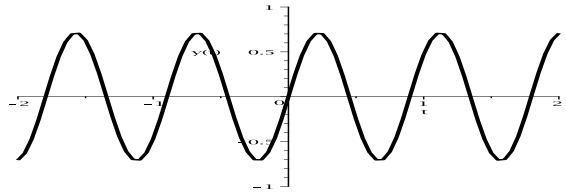
2nd order linear homogenous differential equation with NO damping

$\frac{d^2}{dx^2} y(x) + 49\, y(x) = 0,\ y(0) = 0,\ \mathrm{D}(y)(0) = 5$

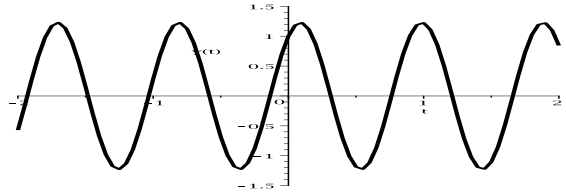
General solution: $y(t) = C1 \sin(7.0\, t) + C2 \cos(7.0\, t)$

IVP solution: $y(t) = 0.7142857143 \sin(7.0\, t)$



Note initial conditions affect long term behaviour when there is NO damping. For example if y(0) = 1 and y'(0) = 5:

IVP solution: $y(t) = 0.7142857143 \sin(7.0\, t) + \cos(7.0\, t)$

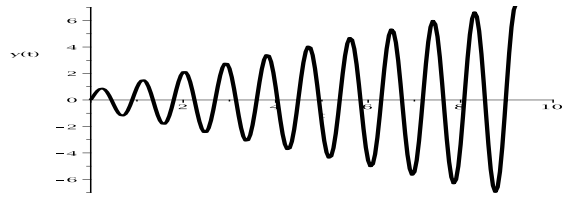


2nd order linear NON-homogenous differential equation with NO damping

$y'' + 49y = 10cos(7t), y(0) = 0.y'(0) = 5$

General solution: $y(t) = \sin(7.0\, t)\ C2 + \cos(7.0\, t)\ C1 + 0.1020408163 \cos(7.0\, t) + 0.7142857143 \sin(7.0\, t)\, t$

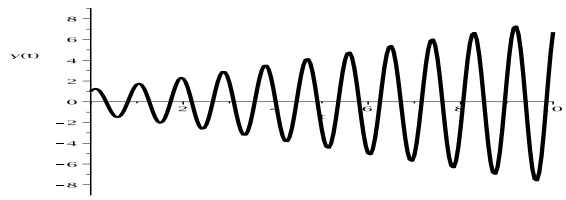
IVP solution: $y(t) = 0.7142857143 \sin(7.0\, t) + 0.7142857143 \sin(7.0\, t)\, t$



Do initial conditions have a long-term effect? **Why?**

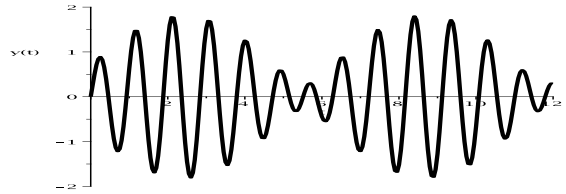
$y'' + 49y = 10cos(7t), y(0) = 1.y'(0) = 5$

IVP solution: $y(t) = 0.7142857143 \sin(7.0\, t) + \cos(7.0\, t) + 0.7142857143 \sin(7.0\, t)\, t$



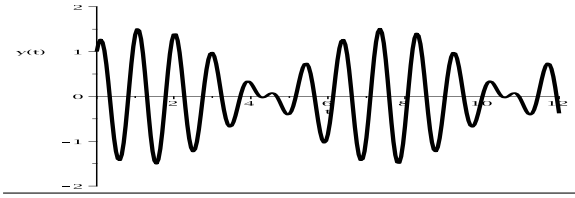
2nd order linear NON-homogeneous differential equation with NO damping

$y'' + 49y = 10cos(6t), y(0) = 0.y'(0) = 5$
General solution: $y(t) = \sin(7.0\,t)\ C2 + \cos(7.0\,t)\ C1 + 0.7692307692\ \cos(6.0\,t)$
IVP solution: $y(t) = 0.7142857143\ \sin(7.0\,t) - 0.07692307692\ \cos(7.0\,t) + 0.07692307692\ \cos(6.0\,t)$



Do initial conditions have a long-term effect? Why?

$y'' + 49y = 10cos(6t), y(0) = 1.y'(0) = 5$
$y(t) = 0.7142857143\ \sin(7.0\,t) + 0.9230769231\ \cos(7.0\,t) + 0.07692307692\ \cos(6.0\,t)$



2nd order homogeneous linear differential equation with DAMPING:

$y'' + y' + 49y = 0$
General solution: $y(t) = C1\ \mathrm{e}^{-0.5000000000\,t}\sin(6.982120020\,t) + C2\ \mathrm{e}^{-0.5000000000\,t}\cos(6.982120020\,t)$

$y'' + 14y' + 49y = 0$
General solution: $y(t) = C1\ \mathrm{e}^{-7.0\,t} + C2\ \mathrm{e}^{-7.0\,t}t$

$y'' + 15y' + 49y = 0$
General solution: $y(t) = C1\ \mathrm{e}^{-4.807417596\,t} + C2\ \mathrm{e}^{-10.19258240\,t}$

$y'' + 14y' + 49y = 0, y(0) = 0, y'(0) = 5$
IVP solution: $y(t) = 5.0\,\mathrm{e}^{-7.0\,t}t$

2nd order NON-homogeneous linear differential equation with DAMPING:

$y'' + 14y' + 49y = 100cos(t), y(0) = 0, y'(0) = 5$
General solution: $y(t) = C2\ \mathrm{e}^{-7.0\,t} + \mathrm{e}^{-7.0\,t}t\,C1 + 1.920000000\ \cos(t) + 0.5600000000\ \sin(t)$
IVP solution: $y(t) = -1.920000000\,\mathrm{e}^{-7.0\,t} - 9.0\,\mathrm{e}^{-7.0\,t}t + 1.920000000\ \cos(t) + 0.5600000000\ \sin(t)$