1.1: Examples of differentiable equation:

1.) $F = ma = m\frac{dv}{dt} = mg - \gamma v$

2.) Mouse population increases at a rate proportional to the current population:

More general model : $\frac{dp}{dt} = rp - k$
where $p(t)$ = mouse population at time t,
r = growth rate or rate constant,
k = predation rate = # mice killed per unit time.

---

direction field = slope field = graph of $\frac{dv}{dt}$ in t, v-plane.

www.math.rutgers.edu/$\sim$ sontag/JODE/JOdeApplet.html

*** can use slope field to determine behavior of v including as $t \to \infty$.

Equilibrium Solution = constant solution

1.2: Solve $\frac{dy}{dt} = ay + b$ by separating variables:

$\frac{dy}{ay+b} = dt$

$\int \frac{dy}{ay+b} = \int dt$ implies $\frac{ln|ay+b|}{a} = t + C$

$ln|ay+b| = at + C$ implies $e^{ln|ay+b|} = e^{at+C}$

$|ay+b| = e^C e^{at}$ implies $ay + b = \pm(e^C e^{at})$

$ay = Ce^{at} - b$ implies $y = Ce^{at} - \frac{b}{a}$

Initial Value Problem: $y(t_0) = y_0$

1.3:

ODE (ordinary differential equation): single independent variable

Ex: $\frac{dy}{dt} = ay + b$

vs

PDE (partial differential equation): several independent variables

Ex: $\frac{\partial xy}{\partial x} = \frac{\partial xy}{\partial y}$

---

order of differential eq'n: order of highest derivative

example of order n: $y^{(n)} = f(t, y, ..., y^{(n-1)})$

Linear vs Non-linear

linear: $a_0(t)y^{(n)} + ... + a_n(t)y = g(t)$

Determine if linear or non-linear:

Ex: $ty'' - t^3y' - 3y = sin(t)$

Ex: $2y'' - 3y' - 3y^2 = 0$

********Existence of a solution**************

********Uniqueness of solution***************

CH 2: Solve $\frac{dy}{dt} = f(t, y)$

2.2: Separation of variables: $N(y)dy = P(t)dt$

2.1: First order linear eqn: $\frac{dy}{dt} + p(t)y = g(t)$

Ex 1: $t^2y' + 2ty = tsin(t)$

Ex 2: $y' = ay + b$

Ex 3: $y' + 3t^2y = t^2$, $y(0) = 0$

Note: could use section 2.2 method, separation of variables to solve ex 2 and 3.

Ex 1: $t^2y' + 2ty = sin(t)$
(note, cannot use separation of variables).

$t^2y' + 2ty = sin(t)$

$(t^2y)' = sin(t)$

$\int(t^2y)'dt = \int sin(t)dt$

$(t^2y) = -cos(t) + C$ implies $y = -t^{-2}cos(t) + Ct^{-2}$

Gen ex: Solve $y' + p(x)y = g(x)$

Let $F(x)$ be an anti-derivative of $p(x)$

$e^{F(x)}y' + [p(x)e^{F(x)}]y = g(x)e^{F(x)}$

$e^{F(x)}y' + [F'(x)e^{F(x)}]y = g(x)e^{F(x)}$

$[e^{F(x)}y]' = g(x)e^{F(x)}$

$e^{F(x)}y = \int g(x)e^{F(x)}dx$

$y = e^{-F(x)}\int g(x)e^{F(x)}dx$