Math 34 Differential Equations Exam #1
September 24, 2010 SHOW ALL WORK

[5] 1a.) Define: A function f is linear if

Circle T for True or F for False:

[3] 1b.) If $y = \phi_1(t)$ and $y = \phi_2(t)$ are solutions to a second order homogeneous differential equation, then $c_1\phi_1 + c_2\phi_2$ is also a solution.

T F

[3] 1c.) If $y = \phi_1(t)$ and $y = \phi_2(t)$ are solutions to a second order linear homogeneous differential equation, then $c_1\phi_1 + c_2\phi_2$ is also a solution.

T F

[3] 1d.) $ln(t)y'' - \frac{y'}{t} + y\sqrt{t} = e^t cos(t)$ is a second order linear differential equation.

T F

[3] 1e.) If p, and g are continuous, then there exists a unique solution to

$$y' + p(t)y = g(t),\ y(0) = 2.$$

T F

[3] 1f.) A first order linear differential equation has a unique solution such that $y(0) = 2$.

T F

Choose 4 problems from problems 2 - 6. You may do all the problems for up to 4 pts extra credit. If you do not choose your best 4 problems, I will substitute your extra problem for your lowest scoring problem, but with a 3 point penalty (if it improves your grade).

Circle the numbers corresponding to your 4 chosen problems: 2 3 4 5 6

Extra credit problem (choose 1 from problems 2 - 6): ____________

[16] 2a.) Match the following differential equation to its direction field. Indicate all equilibrium solutions (if any) and state whether **stable, unstable or semi-stable.** If a differential equation has no equilibrium solutions, state so.

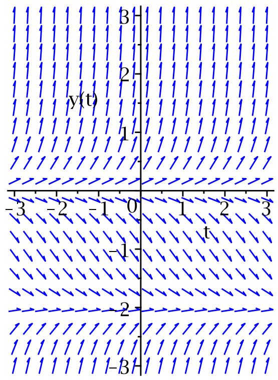
I.) $y' = 1 - y$

II.) $y' = -1 + y$

III.) $y' = y(y + 2)$

IV.) $y' = y^2(2 - y)$

V.) $y' = t + y$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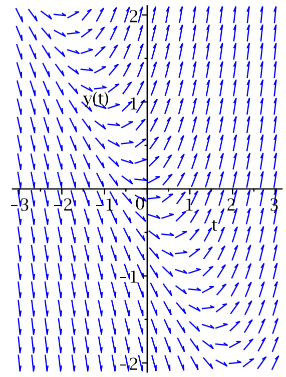
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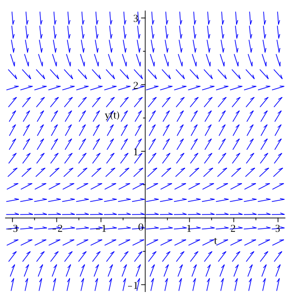
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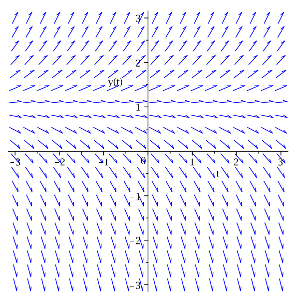
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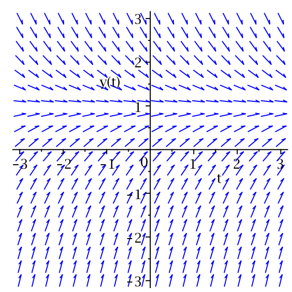

E.)

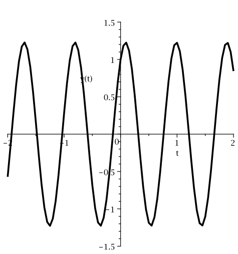
[4] 2b.) Match the following differential equation initial value problem to its graph:

I.) $y'' + y' + 49y = 0$, $y(0) = 0$, $y'(0) = 5$

II.) $y'' + y' + 49y = 0$, $y(0) = 1$, $y'(0) = 5$

III.) $y'' + 49y = 0$, $y(0) = 0$, $y'(0) = 5$

IV.) $y'' + 49y = 0$, $y(0) = 1$, $y'(0) = 5$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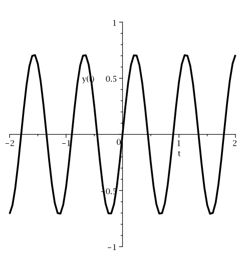
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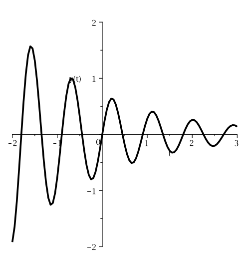
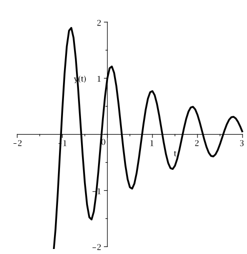

C.)


D.)

3.) Solve the differential equation $t^3y' + 3t^2y = \frac{ln(e)}{t^2-4}$. Simplify your answer.

Answer: ____

4.) Solve $\frac{y''}{y'} - \frac{1}{y^2} = 0, \;\; y(2) = 1, \;\; y'(2) = -1$

Answer: ______________________________

5.) A mass of 10 kg stretches a spring 9.8m. The mass is pushed upward, contracting the spring a distance of one meter and set in motion with an upward velocity of 4 m/sec. If the mass moves in a medium that imparts a viscous force of 100 N when the speed of the mass is 5 m/sec, find the equation of motion of the mass.

Answer: ______________________________

6.) Show that L : set of all twice differentiable functions $\rightarrow$ set of all functions, $L(f) = af'' + bf' + cf$ is a linear function.

Hint: Calculate $L(rf + tg)$ where r, t are real numbers and f, g are twice differentiable functions.

If $y = \phi(t)$ is a solution to $af'' + bf' + cf = 0$, then $L(\phi) =$ ______ .

If $y = \psi(t)$ is a solution to $af'' + bf' + cf = 0$, then $L(\psi) =$ ______.

$L(c_1\phi + c_2\psi) =$ ______ .

Is $c_1\phi + c_2\psi$ a solution to $af'' + bf' + cf = 0$? ________________ .