Calc 1 Review:

Taylor's Thm for $f : \mathbf{R} \to \mathbf{R}$. Suppose $f \in C^k$,

Let $p_k(x) = f(a) + f'(a)(x-a) + ... + \frac{f^{(k)}(a)}{k!}(x-a)^k$

Then $f(x) = p_k(x) + R_k(x,a)$ where $lim_{x \to a} \frac{R_k(x,a)}{(x-a)^k} = 0$.

Prop 1.2: If $f^{(k+1)}$ exists, then there exists c between a and x such that $R_k(x,a) = \frac{f^{(k+1)}(c)}{(k+1)!}(x-a)^{k+1}$

Estimate $ln(2)$ using degree 3 Taylor polynomial for $f(x) = ln(x)$ about $a = 1$

$f(x) = ln(x) \qquad f'(x) = x^{-1} \qquad f''(x) = -x^{-2} \qquad f'''(x) = 2x^{-3}$

$p_3(x) =$

Thus $ln(2) \sim p_3(2) =$

$ln(x) = p_3(x) + R_3(x-1)$

$f^{(4)}(x) = -6x^{-4} = -\frac{6}{x^4}$

$R_3(x-1) = \frac{f^{(4)}(c)}{(4)!}(x-1)^4 =$

where c is btwn 1 and 2.

$ln(2) = p_3(2) + R_3(2,1) =$ where $c \in (1,2)$

Multivariable version:

Taylor's Thm for $f : \mathbf{R} \to \mathbf{R}$. Suppose $f \in C^k$,

Let $p_k(\mathbf{x}) = f(\mathbf{a}) + \Sigma_{i=1}^n f_{x_i}(\mathbf{a})(x_i - a_i) + ...$

$$+\frac{1}{k!}\Sigma_{i_1,...,i_k=1}^n f_{i_1...i_k}(\mathbf{a})(x_{i_1} - a_{i_1})...(x_{i_k} - a_{i_k})$$

Then $f(\mathbf{x}) = p_k(\mathbf{x}) + R_k(\mathbf{x}, \mathbf{a})$ where $lim_{x \to a} \frac{R_k(x,a)}{||x-a||^k} = 0$.

If $f : \mathbf{R}^n \to \mathbf{R} \in C^2$, then

$R_1(\mathbf{x}, \mathbf{a}) = \frac{1}{2}\Sigma_{i,j=1}^n f_{x_i x_j}(\mathbf{c})(x_i - a_i)(x_j - a_j)$

for some $\mathbf{c}$ on the line segment joining $\mathbf{a}$ and $\mathbf{x}$.

If $f : \mathbf{R}^n \to \mathbf{R} \in C^{k+1}$, then

$R_k(\mathbf{x}, \mathbf{a}) = \frac{1}{(k+1)!}\Sigma_{i_1,...,i_{k+1}=1}^n f_{x_{i_1}...x_{i_{k+1}}}(\mathbf{c})(x_{i_1} - a_{i_1})...(x_{i_{k+1}} - a_{i_{k+1}})$

for some $\mathbf{c}$ on the line segment joining $\mathbf{a}$ and $\mathbf{x}$.