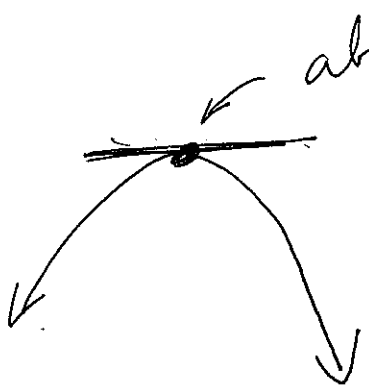
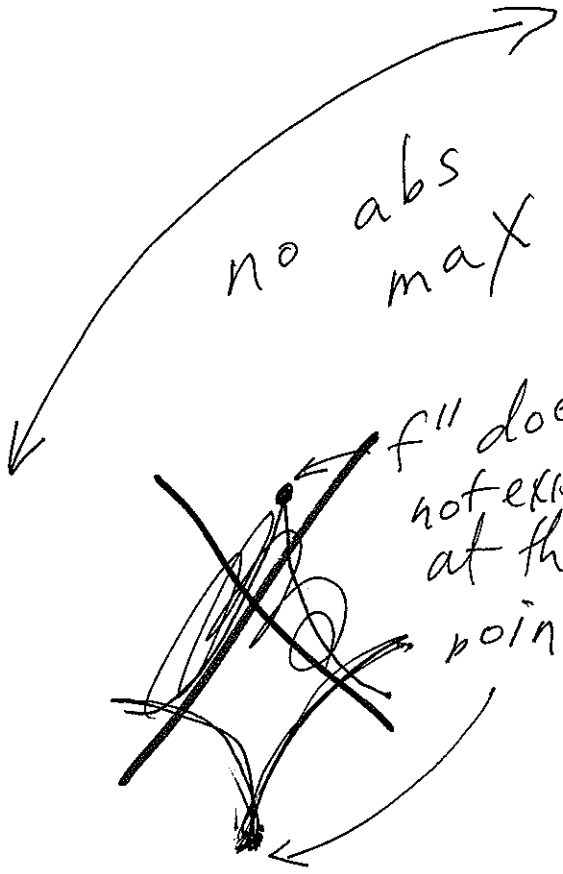
Other methods
(For example if you don't have continuous ~~fon~~ function on a closed interval)
(— or finding 2nd derivative easy)

— analyze the graph

Special cases

1) $f''(x) < 0$ for all x in $\mathbb{R}$



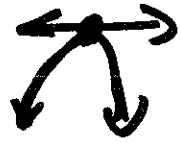


2)

Thm 9: If $f'(x)$ exists for all x in interval I and if $f'(c) = 0$ & c is the only critical point in I

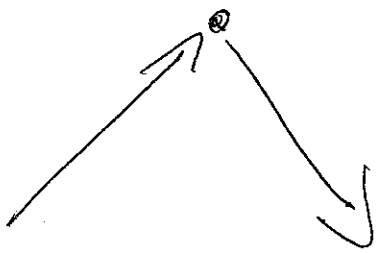
then

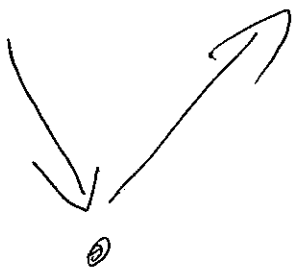
$f''(c) < 0 \Rightarrow f(c)$ abs max



$f''(c) > 0 \Rightarrow f(c)$ abs min

Other method sp: use 1st derivative (special cases)





3.5) Applications of abs max/min

Ex 1 Find 2 numbers whose sum is 100 that have maximum product

1.) What are you maximizing

product = $\boxed{P = xy}$

2.) Turn into a Calc 1 problem by eliminating all but one variable by RHS of eqn

Need eqn that relates x & y

to eliminate y

$$x + y = 100$$

$$y = 100 - x$$

Calc 1 problem:

Maximize $P(x) = x(100 - x)$

3) Solve Calc 1 problem

Maximize $P(x) = 100x - x^2$

Note no restrictions on x

3a) Find critical pts

$P'(x) = 100 - 2x = 0$, DNE

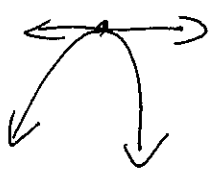
$100 = 2x$

$x = 50$

3b) Determine if abs max at $x = 50$

i) Can't use EVT — no restrictions on x, ie don't have closed interval

ii) $P''(x) = -2 < 0$

By analyzing graph  abs max at $x = 50$

iii) By Thm 9, we note

$P'(x) = 100 - 2x$ exists for all x

$P'(50) = 0$

50 is the only critical pt of P

$P''(50) = -2 < 0$

$\Rightarrow$ abs max at $x = 50$

by Thm 9

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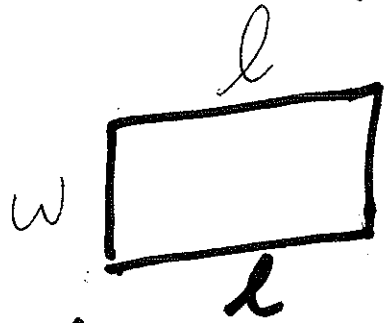
4) State sol'n

$x = 50 \quad y = 100 - 50 = 50$

Two #'s are 50, 50

EX2) Suppose you have 200 m of fencing. What is the largest rectangular area you can fence in? What are the dimensions

0) Draw picture



1) What are we maximizing?

$$A = lw$$

2) Eliminate variables via substitution

Need eqn relating l & w so I can eliminate one of them

Perimeter = $200 = 2l + 2w$

$$100 = l + w$$

$$l = 100 - w$$

$$A(w) = (100 - w)\, w$$

make closed for convenience

3) Maximize

$$A(w) = 100w - w^2$$

where $w \in [0, 100]$

3a) Find critical pts

$$A'(w) = 100 - 2w$$

$$\Rightarrow w = 50$$

3b) Determine if abs max at $w = 50$

i) EVT

w	$A(w) = (100 - w)\, w$
50	$50 \cdot 50 = 2500$
0	0
100	0

abs max occurs at $x = 50$

ii) $f''(x) = -2 < 0$ ✗ for all x

$\Rightarrow f'(50) = 0$

(ii) Thm 9

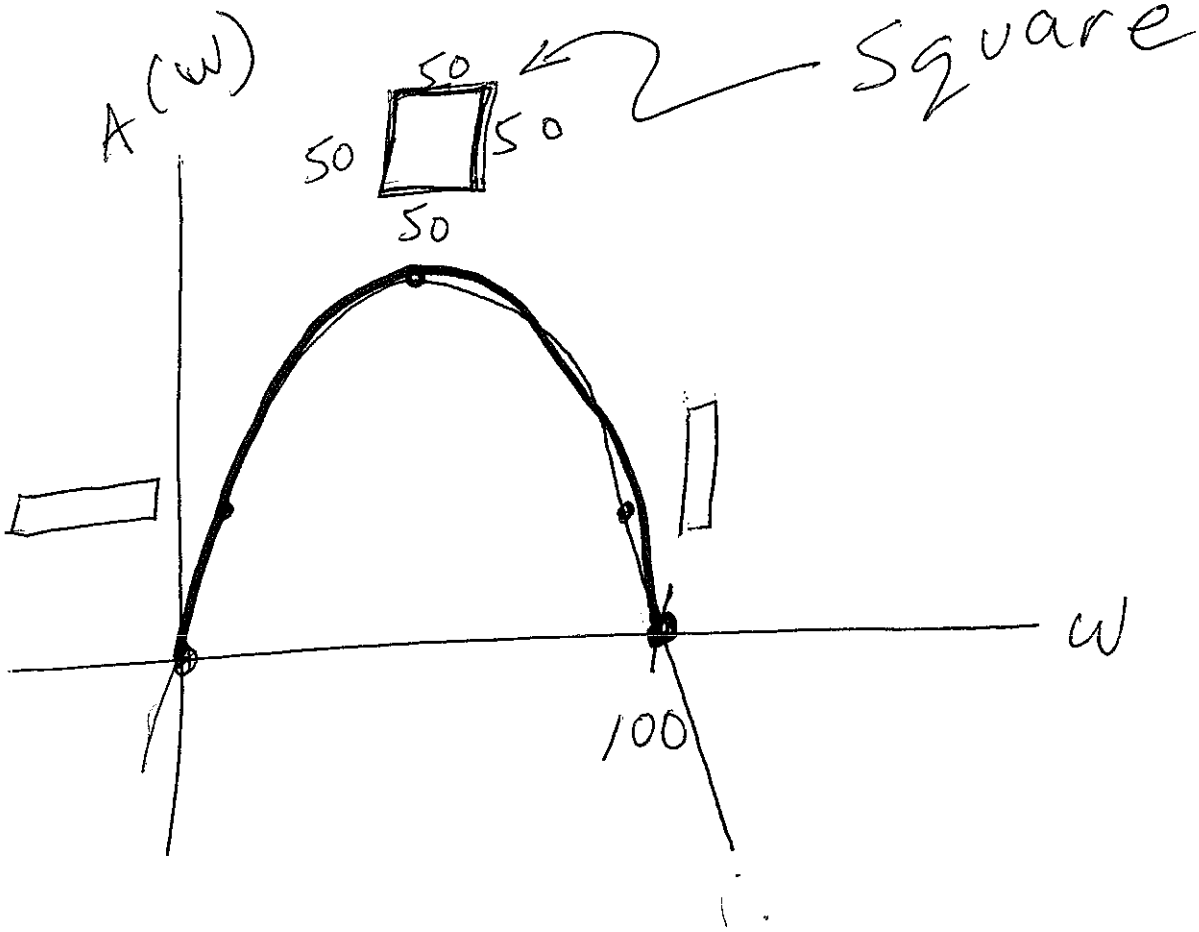
4) State Answer

Largest area $= 2500\ m^2$

Dimensions : $50m \times 50m$

$x = 50$

$y = 100 - 50 = 50$


A(w)
50
50
50
50
Square
w
100