

tangent line at $x = 0$ to $y = (x+3)/(4x+1) + 2$

Input interpretation:
 tangent line to $y = \frac{x+3}{4x+1} + 2$ at $x = 0$

Result:
 $y = 5 - 11x$

Plot:

(x from -0.4 to 0.4)
 — $y = \frac{x+3}{4x+1} + 2$
 — tangent

Computed by Wolfram Mathematica

↙ slope = $f'(0)$
 pt on line = $(0, f(0))$

$$f(x) = \frac{x+3}{4x+1} + 2 \quad \left. \vphantom{f(x)} \right\} \leftarrow f$$

$$f'(x) = \frac{-11}{(4x+1)^2} \quad \left. \vphantom{f'(x)} \right\} \leftarrow \text{derivative}$$

f'
 ↙ function

2.5 Notation

$$y = f(x)$$

slope of tangent line to

$y = f(x)$ at x is

$$f'(x) = y'(x) = \frac{dy}{dx}(x) = \frac{df}{dx}(x)$$

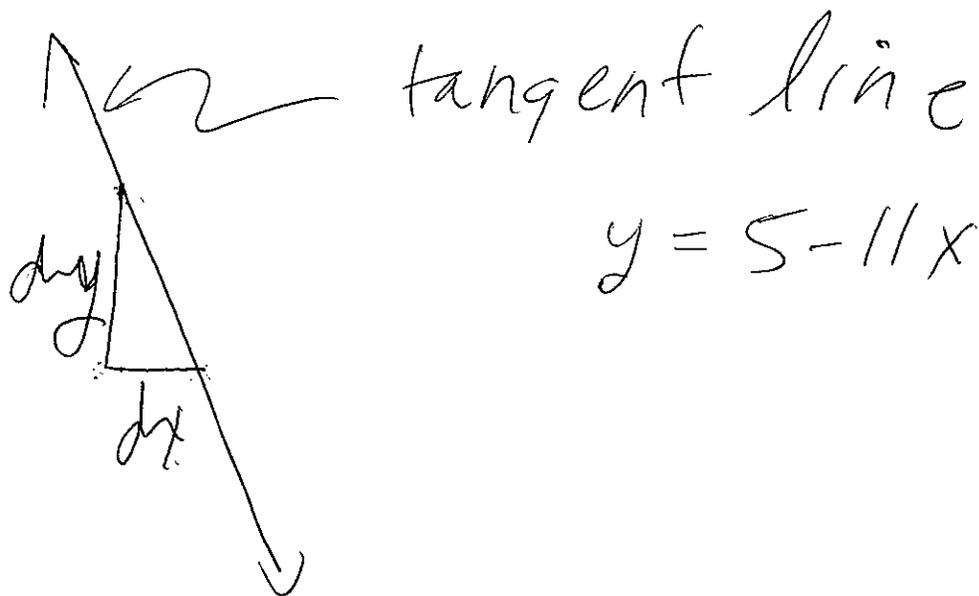
derivative evaluated at x { all mean same thing
slope of tangent line **at x**
instantaneous rate of change

$$f' = y' = \frac{dy}{dx} = \frac{df}{dx}$$

functions { all mean same thing
derivative

$$\text{Slope} = f'(0) = -11 = \frac{dy}{dx} \quad \left(= \frac{dy}{dx}(0) \right)$$

$$\frac{dy}{dx} = -11 \Rightarrow dy = -11dx$$



$$\frac{dy}{dx} \neq \frac{y}{x}$$

$$\frac{\sin(y)}{\sin(x)} \neq \frac{y}{x}$$

tangent line at $x = -1$ to $y = \frac{x+3}{4x+1} + 2$ - WolframAlpha - Windows Internet Explorer

http://www.wolframalpha.com/input/?i=tangent+line+at+x=-1+to+y=%3D+%28x%2B3%29%2F%284x%2B1%29+%2B2

Examples Random

Input interpretation:

tangent line to $y = \frac{x+3}{4x+1} + 2$ at $x = -1$

Result:

$y = \frac{1}{9} - \frac{11x}{9}$

Approximate form:

slope = $f'(-1)$
 point on line $(-1, f(-1))$

Plot:

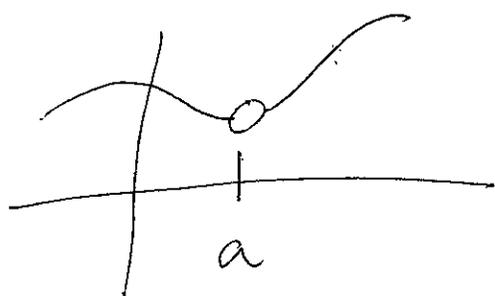
(x from -2 to 0)
 $y = \frac{x+3}{4x+1}$
 tangent

to approximate $f(-1.001)$, can use tangent line to f at $x = -1$

2.4: When is a function

NOT differentiable at

Ex: $f(a)$ is not defined

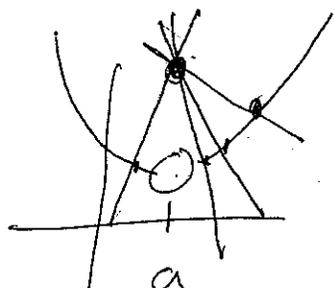


$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - \underbrace{f(a)}}{h}$$

Not diff
at a

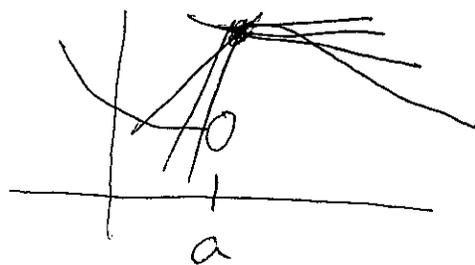
not defined
limit DNE

Ex: f is not continuous at a



$f'(a)$ DNE

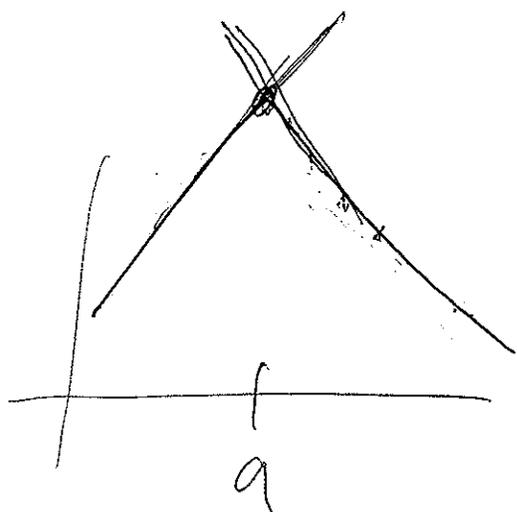
tangent lines



$f'(a)$ DNE

can not be vertical (no ∞ slope)

EX: Sharp corners or cusps



$f'(a)$ DNE



derivative
DNE at a

2.5: formulas for finding derivative

$$\frac{d}{dx} (x^n) = \lim_{h \rightarrow 0} \frac{(x+h)^n - x^n}{h}$$

$f(x) = x^n$
Find $f'(x)$

$y = x^n$
Find y'

$$= n x^{n-1}$$

↑ skip lots of steps

works for all real #'s

not just positive integers

If n is an integer

$$\frac{(x+h)^n - x^n}{h} = \frac{\cancel{x^n} + n x^{n-1} h + \dots + h^n - \cancel{x^n}}{h}$$

$$\text{Ex: } f(x) = x^{0.7}$$

$$f'(x) = 0.7 x^{0.7-1}$$

$$= \boxed{0.7 x^{-0.3}}$$

$$\text{Ex } (x^\pi)' = \pi x^{\pi-1}$$

$$\frac{d}{dx}(\sin x) = \lim_{h \rightarrow 0} \left[\frac{\sin(x+h) - \sin x}{h} \right]$$

$$= \cos(x)$$

↑
skip steps

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx} (c f(x)) = \lim_{h \rightarrow 0} \frac{c f(x+h) - c f(x)}{h}$$

constant

$$= c \left[\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \right]$$

$$= c f'(x)$$

$$\text{Ex: } [5x^2]' = 5(x^2)'$$

$$= 5(2x)$$

$$= 10x$$

$$\text{Ex } (f+g)' = f' + g'$$