log - log plots

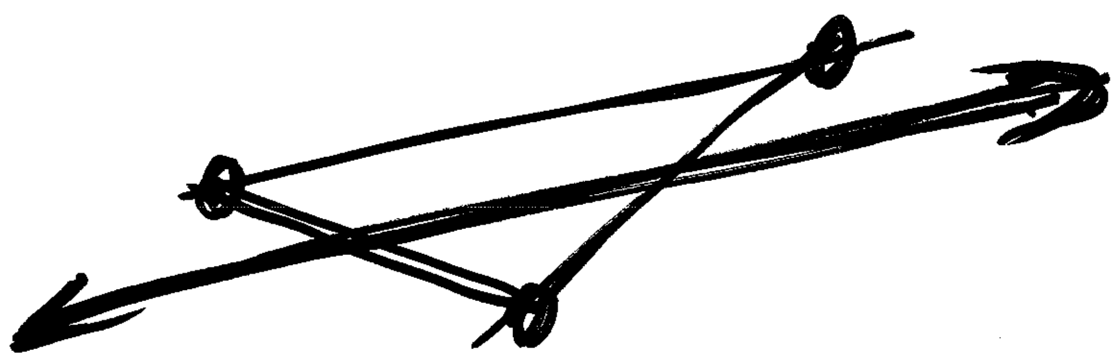
$z = f(t)$

determine f

Let $x = \log t$; $y = \log z$

Suppose $y = mx + b$

$\log z = m \log t + b$



$$y = mx + b$$

$$\log z = m \log t + b$$

$$10^{\log z} = 10^{m \log t + b}$$

$$z = \left(10^{m \log t}\right)\left(10^{b}\right)$$

$$z = \left(10^{\log t^{m}}\right)\left(10^{b}\right)$$

$$z = \left(10^{b}\right) t^{m}$$

$$z = A t^{m}$$

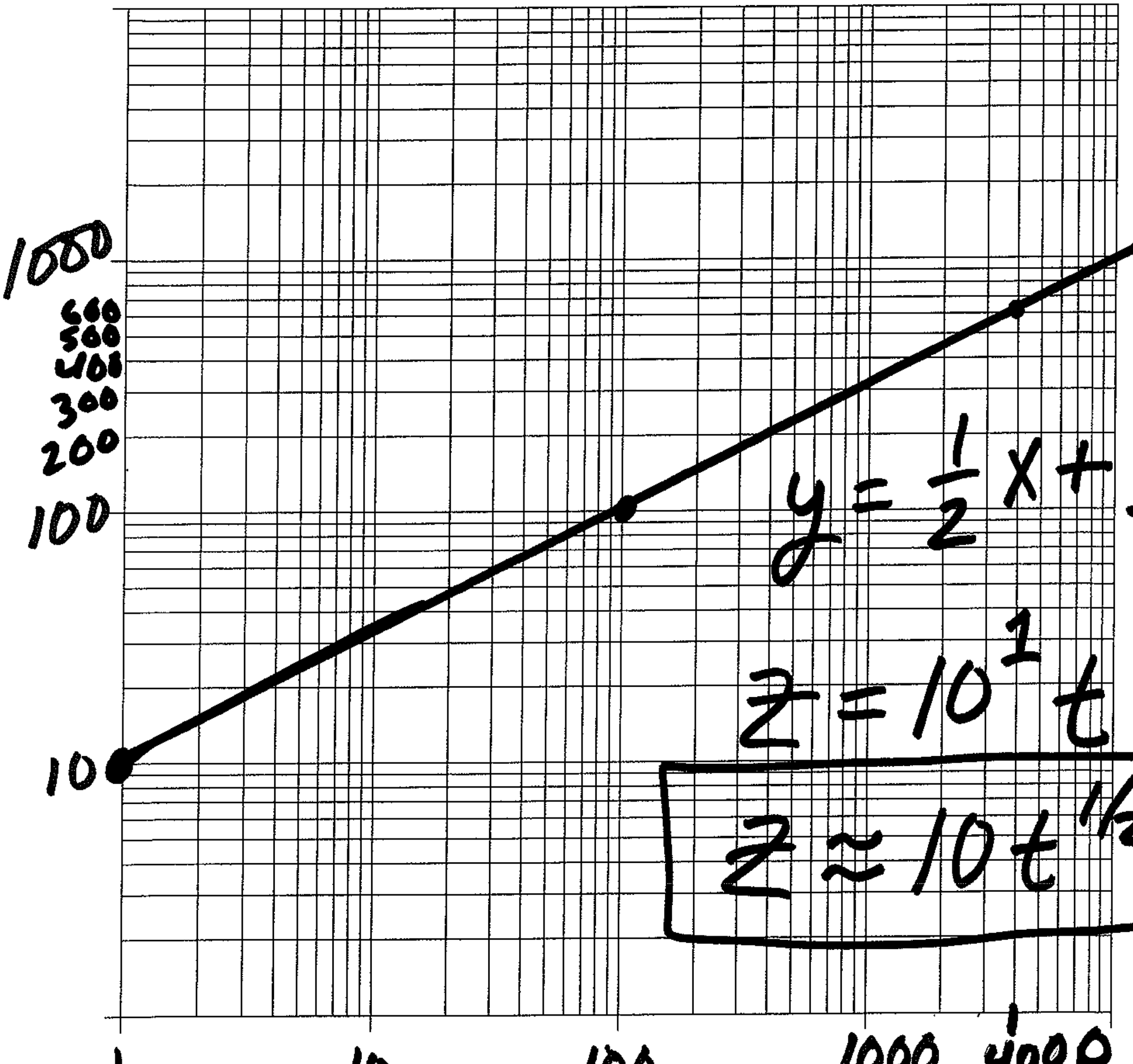
$$A = 10^{b}$$ ← y-intercept of line $y = mx + b$

m = slope

t	z
1	10
100	100
4000	632.5

$\rightarrow$

$\log t = x$	$y = \log z$
0	1
2	2
$\log(4000)$	$\log(632.5)$



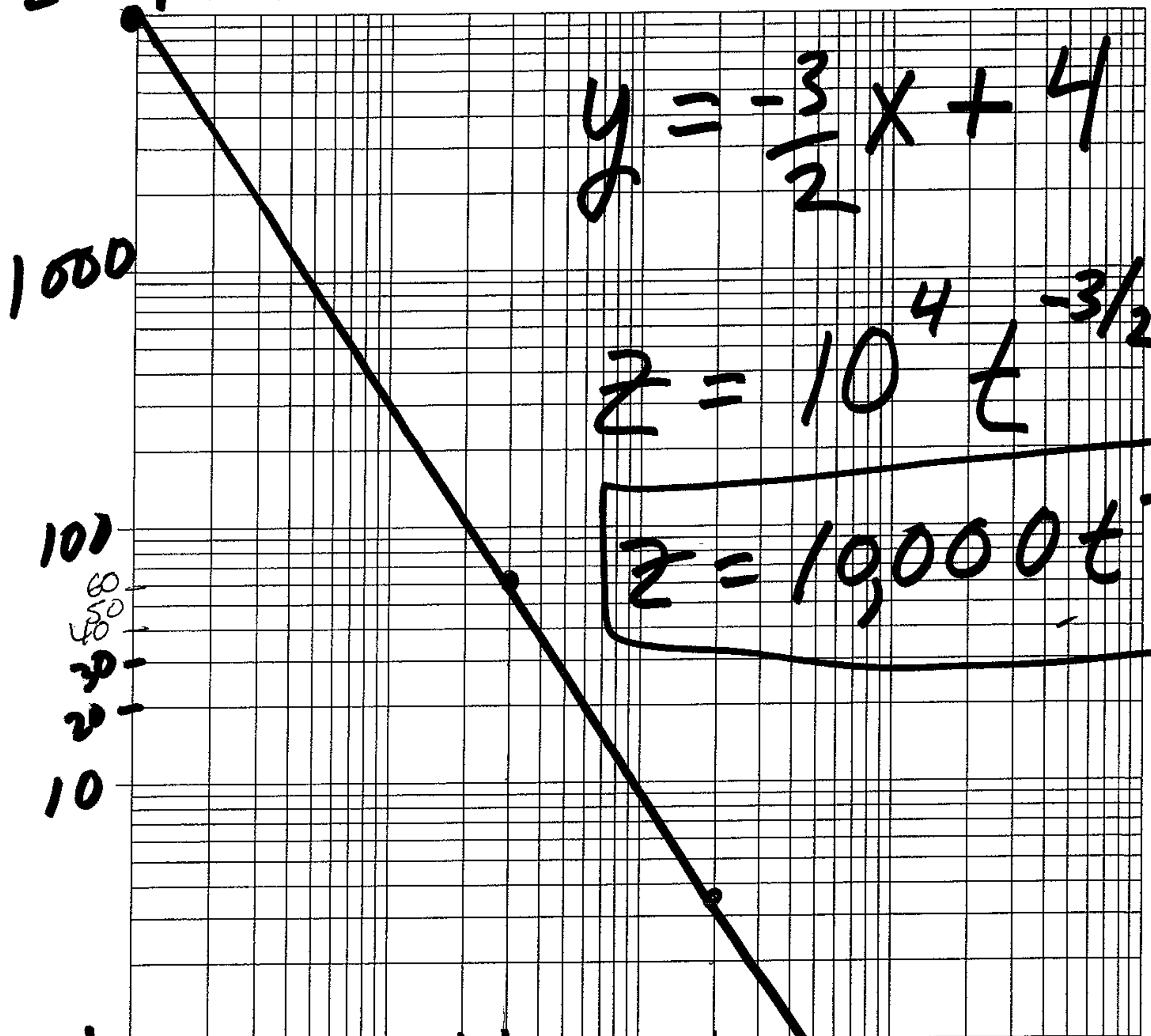
$$y = \frac{1}{2}x + 1$$

$$z = 10^{1} t^{1/2}$$

$$\boxed{z \approx 10t^{1/2}}$$

z	$?$

t	z
1	10,000
30	60
200	3.5



$$y = -\frac{3}{2}x + 4$$

$$z = 10^4 t^{-3/2}$$

$$\boxed{z = 10{,}000\, t^{-3/2}}$$

semi-log paper

x	y
0	1
1	100
2	10,000

$\longrightarrow$

t=	z
0	0
1	2
2	4

t=x   z=log y

Let $x = t$; $y = \log z$

Suppose $y = mx + b$

$$\log z = mt + b$$

$$10^{\log z} = 10^{mt+b}$$

$$z = (10^b)(10^m)^t$$

$$= Ad^t$$

semi-log paper

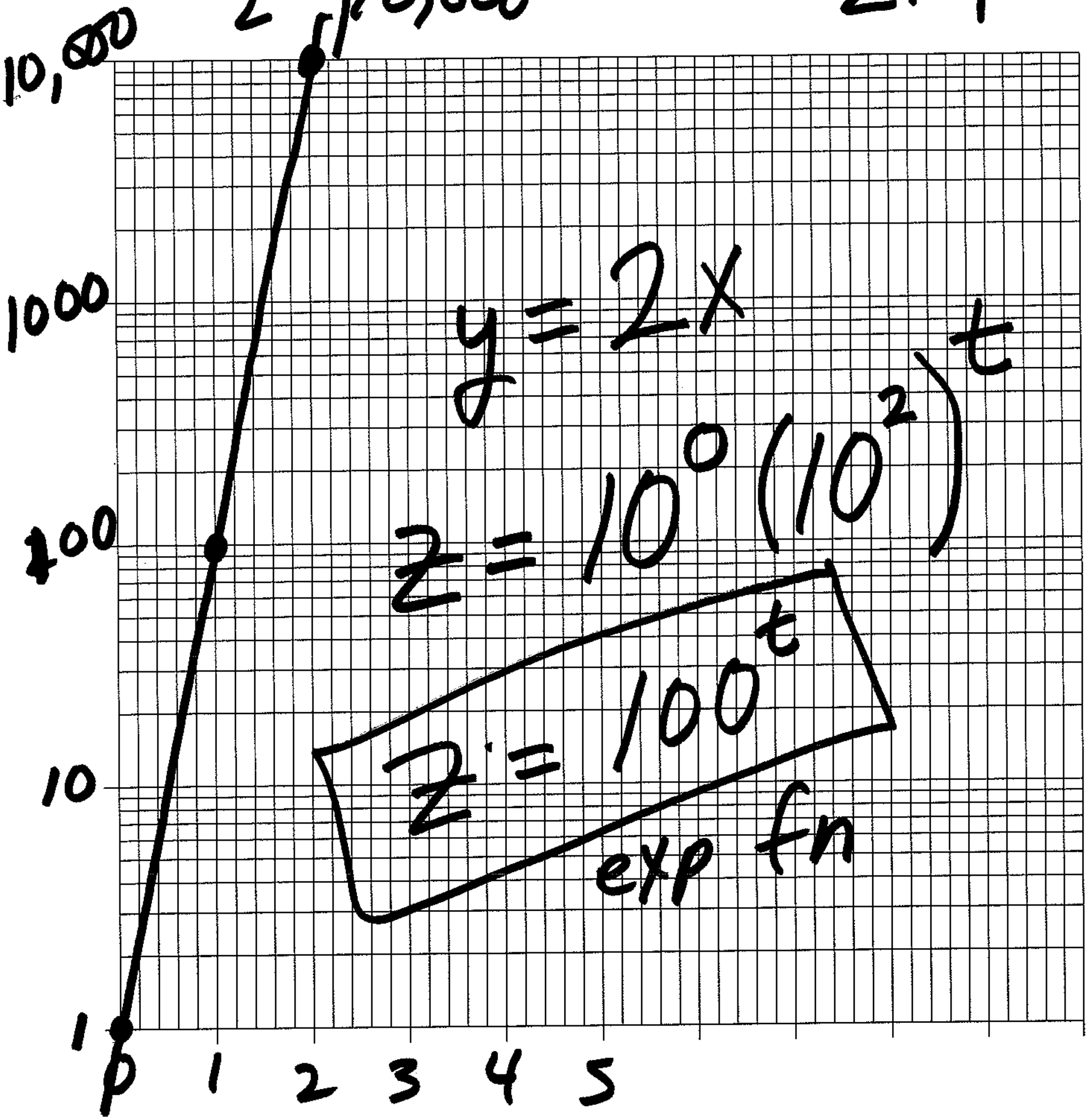
$\Updownarrow$

exponential growth/decay

t	z
0	1
1	100
2	10,000

$\longrightarrow$

$t = x$	$y = \log z$
0	0
1	2
2	4



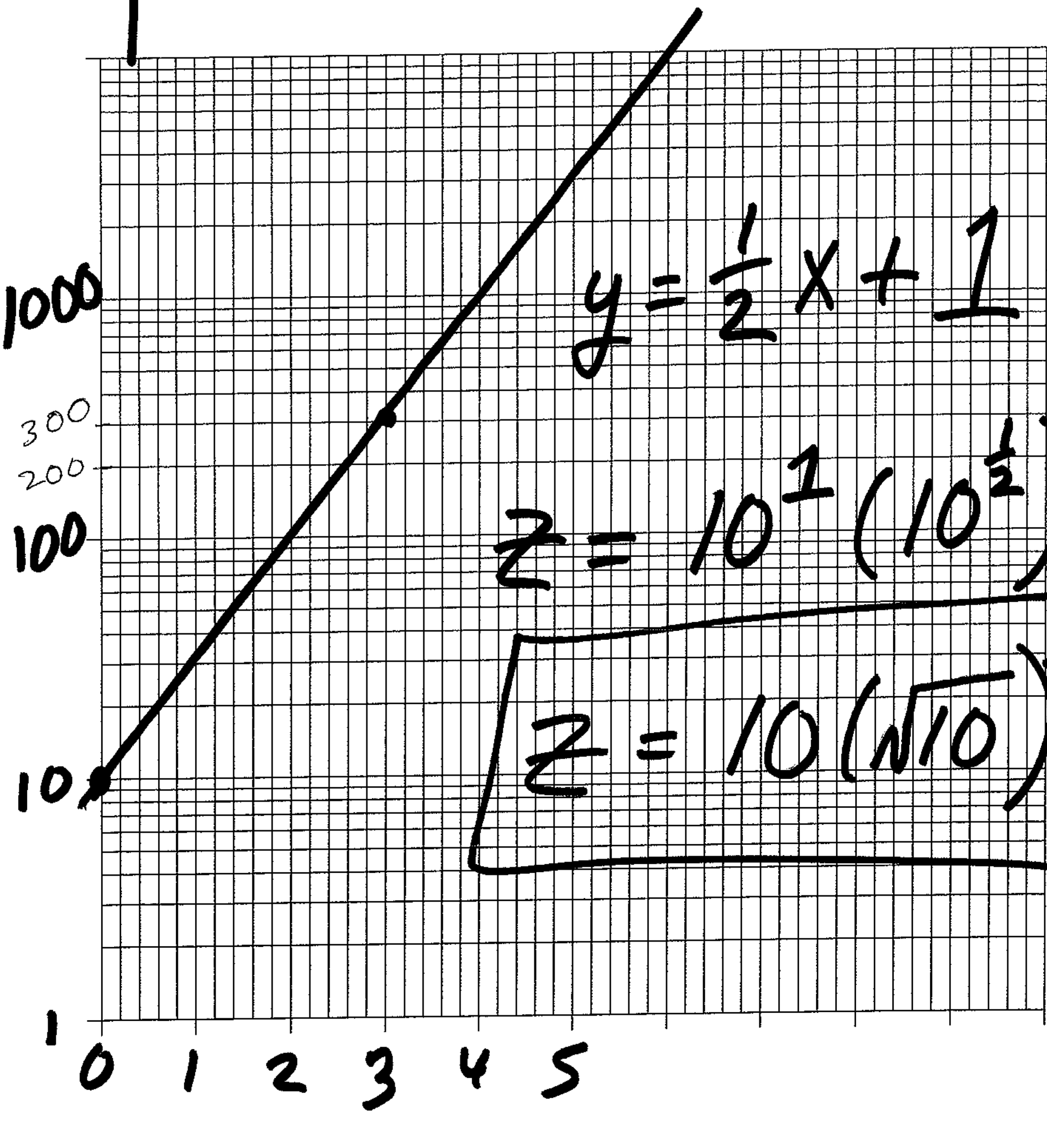
$$y = 2x$$

$$z = 10^0 \left(10^2\right)^t$$

$$\boxed{z = 100^t}$$

exp fn

t	z
0	10
3	316



$$y = \frac{1}{2}x + 1$$

$$z = 10^{1}\left(10^{\frac{1}{2}}\right)^{t}$$

$$\boxed{z = 10\left(\sqrt{10}\right)^{t}}$$

Section 4.3/4.4 Exponential growth/decay

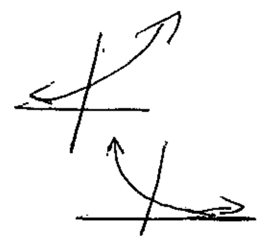
Thm 8: Suppose c, k are constants. Then

$$\frac{dy}{dx} = ky \text{ if and only if } \boxed{y = ce^{kx}}$$

I.e., If the (instantaneous) rate of change of y with respect to x is proportional to y, then

Section 4.3: $k > 0$ implies exponential growth.

Section 4.4: $k < 0$ implies exponential decay.

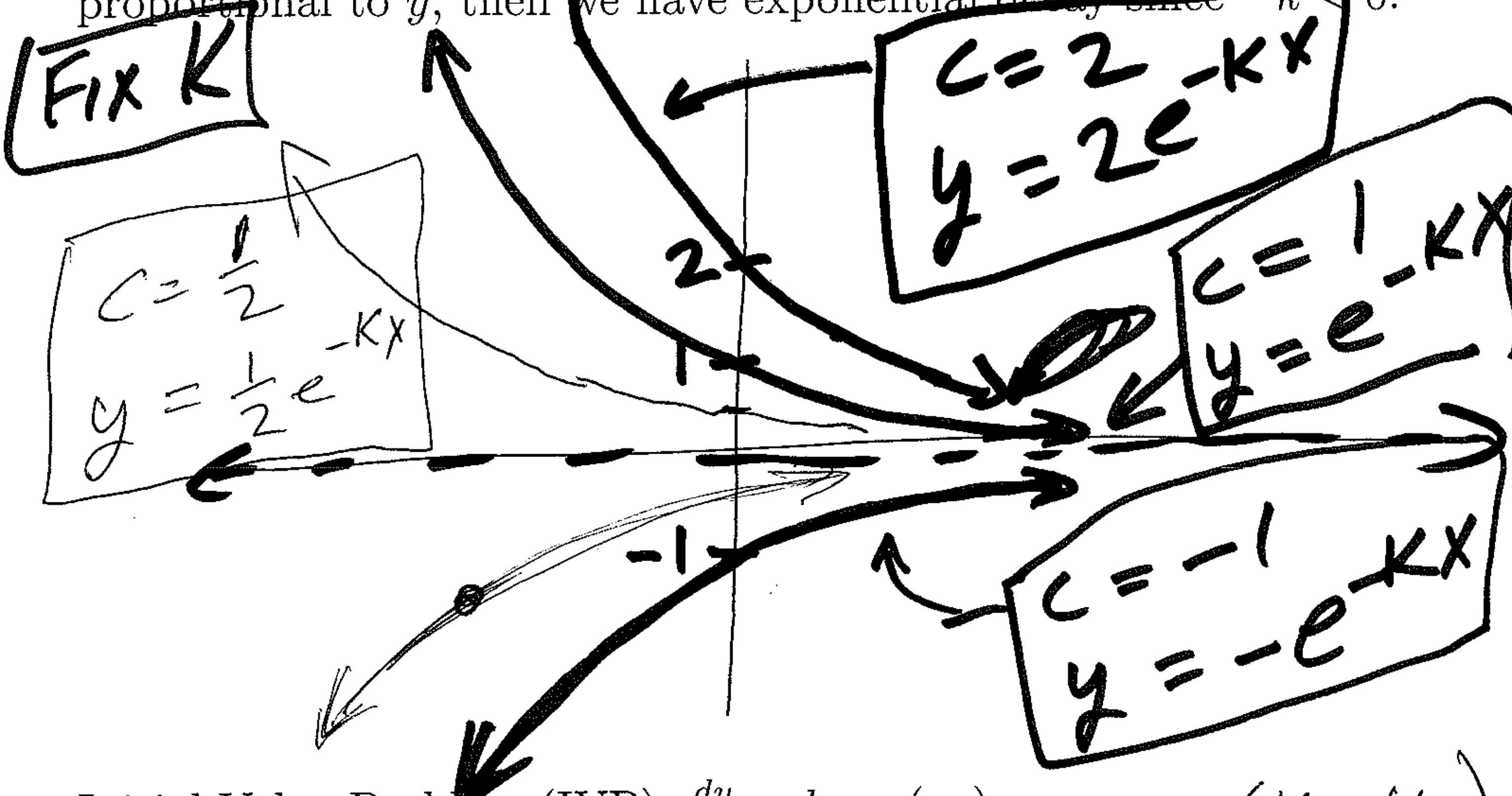


For simplicity, take $k > 0$. Then in section 4.4,

Section 4.4 version of Thm 8: Suppose c, k are constants. Then

$$\frac{dy}{dx} = -ky \text{ if and only if } \boxed{y = ce^{-kx}}$$

I.e., If the (instantaneous) rate of change of y with respect to x is proportional to y, then we have exponential decay since $-k < 0$.



Initial Value Problem (IVP): $\frac{dy}{dx} = ky$, $y(x_0) = y_0 \leftarrow (x_0, y_0)$

if and only if $y = ce^{kx}$, $y_0 = ce^{kx_0} \Rightarrow c = \frac{y_0}{e^{kx_0}}$

Hence $y = ce^{kx}$ where $c = \frac{y_0}{e^{kx_0}} \leftarrow$ unique soln to IVP

Precalculus:

Let $P(0) = P_0$

Section 4.3, $k > 0$**:**

Doubling time = generation time: If $P(t) = P_0 e^{kt}$, then at what time t is $P(t) = 2P_0$

$$2\cancel{P_0} = \cancel{P_0}e^{kt} \Rightarrow \boxed{2 = e^{kt}} \Rightarrow ln(2) = ln(e^{kt}) = kt \Rightarrow t = \frac{ln(2)}{k} \approx \frac{0.69}{k}$$

Section 4.4, $-k < 0$**:**

Half life: If $P(t) = P_0 e^{-kt}$, then at what time t is $P(t) = \frac{1}{2}P_0$

$$\frac{1}{2}\cancel{P_0} = \cancel{P_0}e^{-kt} \Rightarrow \frac{1}{2} = e^{-kt}$$ take reciprical

$$\Rightarrow \boxed{2 = e^{kt}} \Rightarrow ln(2) = ln(e^{kt}) = kt \Rightarrow \boxed{t = \frac{ln(2)}{k} \approx \frac{0.69}{k}}$$

You do NOT need to know the following for either exam 1b or exam 2:

Thm 11: Newton's law of cooling

The rate of change of temperature T with respect to time t is given by

$$\boxed{\frac{dT}{dt} = -k(T - S)} \Leftarrow \text{Ch 8}$$

where $k > 0$ is the proportionality constant, and S is the constant temperature of the surrounding medium.

Hence $T(t) = P_0 e^{-kt} + S$ where $P_0 = T(0) - C$

anti derivative

Proof: $T' = -k(T - S) \Rightarrow \frac{T'}{T-S} = -k \Rightarrow ln|T - S| = -kt + C_1$, for some constant C_1

derivative

$|T - S| = e^{ln|T-S|} = e^{-kt+C_1} = e^{-kt}e^{C_1}$

Hence $T - S = C_2 e^{-kt}$ for some constant C_2

When $t = 0$, ~~$T(0) - S = C_2 e^0 = C_2$~~

Hence $\boxed{T(t) = P_0 e^{-kt} + S \text{ where } P_0 = T(0) - C}$

5.1)

ch 4: $(t^2)' \rightarrow 2t$

$$(t^2+5)' = 2t$$

$$(t^2+c)' = 2t$$

ch 5: $\int 2t\,dt = t^2 + C$

derivative

$$\int t^3\,dt = \frac{t^4}{4} + C$$

anti derivative