Math 150 Exam 1
October 4, 2006

Choose 7 from the following 9 problems. Circle your choices: 1 2 3 4 5 6 7 8 9
You may do more than 7 problems in which case your two unchosen problems can replace your lowest one or two problems at 2/3 the value as discussed in class.

1.) P(10, 7) =________

$C(10,7) = \binom{10}{7} =$________

The inversion sequence for the permutation 15243 is ____________________

The permutation corresponding to the inversion sequence 3, 0, 2, 1, 0 is ____________________

2.) $r(9,2) =$____________________

$r(3,3) =$____________________

Given that $\{x_{13}, x_{12}, x_7, x_1\}$ is a 4-combination of $\{x_{13}, x_{12}, ..., x_1, x_0\}$, Determine the combinations which come immediately before and after the combination $\{x_{13}, x_{12}, x_7, x_1\}$, using the base 2 generating scheme.

Before $\{x_{13}, x_{12}, x_7, x_1\}$: ______________________________

After $\{x_{13}, x_{12}, x_7, x_1\}$: ______________________________

Determine the 4-combinations of $\{1, 2, ..., 14\}$ which come immediately before and after the the 4-combination $\{2, 8, 13, 14\}$ in lexicographical ordering.

Before $\{2, 8, 13, 14\}$: ______________________________

After $\{2, 8, 13, 14\}$:______________________________

3.) In how many ways can 9 indistinguishable rooks be places on a 20-by-20 chessboard so that no rook can attack another rook?

In how many ways can 9 rooks be places on a 20-by-20 chessboard so that no rook can attack another rook if no two rooks have the same color?

4.) How many different circular permutations can be made using using 30 beads if you have 20 green beads, 9 blue beads and 1 red beads?

5.) How many sets of 3 numbers each can be formed from the numbers $\{1, 2, 3, ..., 50\}$ if no two consecutive numbers are to be in a set?

6.) Use the pigeonhole principle to prove that in a group of n people where $n > 1$, there are at least 2 people who have the same number of acquaintances. State where you use the pigeonhole principle.

7.) Suppose $x, y \in \mathcal{Z}$. Define a relation on $\mathcal{Z}$ such that $x \sim y$ iff there exists $k \in \mathcal{Z}$ such that $x - y = 5k$. Show $\sim$ is an equivalence relation on $\mathcal{Z}$. What are the equivalence classes?

8.) Let $X = \{1, 2, 3\}$. Define a partial order on $X \times X$ by $(x_1, y_1) \leq_x (x_2, y_2)$ iff $x_1 \leq x_2$ (for example $(1, 3) \leq_x (2, 1)$). Is $\leq_x$ reflexive? Is $\leq_x$ symmetric? Is $\leq_x$ antisymmetric? Is $\leq_x$ transitive? Is $\leq_x$ a partial order? Is $\leq_x$ an equivalence relation? Give a proof for each answer.

9.) Use a combinatorial argument to prove $\Sigma_{k=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix} \begin{pmatrix} n \\ n-k \end{pmatrix} = \begin{pmatrix} 2n \\ n \end{pmatrix}$