AND: multiply

OR: add

Permutations: order matters

Combinations: order does NOT matter

Quotient Rule/Identifications/Multisets

Inclusion/Exclusion

Recursion (generating functions/exponential generating functions)

Defn: The *Catalan sequence* $C_n = \frac{1}{n+1} \binom{2n}{n}$

Thm 8.1.1
Let $S_n = \{a_1, a_2, ..., a_{2n} \mid a_1, a_2, ..., a_{2n}$ is a sequence containing n +1's and n -1's such that $\Sigma_{i=0}^{k} a_k \geq 0 \ \forall k = 1, 2, ..., 2n\}$. Then $|S_n| = C_n$

Ex:

$|S_1| = |\{1, -1\}| = 1 = C_1 = \frac{1}{2} \binom{2}{1}$

$|S_2| = |\{1, 1, -1, -1; \ 1, -1, 1, -1\}| = 2 = C_2 = \frac{1}{3} \binom{4}{2}$

$|S_3| = |\{1, 1, 1, -1, -1, -1; \ 1, 1, -1, 1, -1, -1; \ 1, 1, -1, -1, 1, -1;$
$1, -1, 1, 1, -1, -1; \ 1, -1, 1, -1, 1, -1\}| = 5 = C_3 = \frac{1}{4} \binom{6}{3}$