Mathematics 90 Final Exam – F. Goodman
December 1998

1. **(a)** How many ways are there to distribute 3 identical oranges, 5 identical candy canes, and one lump of coal to 3 (distinguishable) holiday stockings?

(b) What if the candy canes are of five distinct flavors?

2. How many integer solutions are there to:

(a) $x_1 + x_2 + x_3 + x_4 = 15,\ \ x_i \geq 0$?

(b) $x_1 + x_2 + x_3 + x_4 \leq 15,\ \ x_i \geq 0$?

(c) $x_1 + x_2 + x_3 + x_4 = 15,\ \ x_i \geq 0,\ x_2 \geq 2,\ x_4 \leq 3$?

3. Give generating function models for each of the parts of problem 2.

4. A party of n English tourists check their umbrellas at the Loony Toons amusement park in Los Angeles. The rabbit in charge of the hatcheck room returns the umbrellas randomly. How many arrangements of umbrellas are there in which no person receives his own umbrella? Use inclusion-exclusion. Check your work by considering small values of n.

5. Cadilacs, Oldsmobile PLR's,and Honda Civics are parked along a street. Each Cadilac and Oldsmobile requires two parking places while a Honda requires only one. Let a_r be the number of arrangements of cars in r parking places (with no empty places between cars). Find a recurrance relation for a_r and solve to find a formula for a_r. What is the limit $\lim_{r\to\infty} \frac{a_{r+1}}{a_r}$? Use the recurrance relation to find the generating function $g(x) = \sum_{r=0}^{\infty} a_r x^r$.