

Mathematics 90 – Hour Exam
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General instructions: Explain your reasoning for full (and partial) credit.

1. Find the generating function for the number of ways to make r cents in change using pennies, dimes, and quarters. What is the coefficient of x^{60} ?
2. How many ways are there to choose 10 coins from a collection of pennies, nickels and dimes (at least 10 of each)? What is the generating function for the number of ways to choose r coins from a limitless supply of pennies, nickels, and dimes?
3. Use a block-walking argument (or other correct argument of your choice) to show that:

$$\sum_{k=0}^m \binom{m}{k} \binom{n}{r-k} = \binom{m+n}{r}$$

Apply this result to show that

$$\sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n}$$