Mathematics 90 Midterm Exam I – F. Goodman
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Version 1

1. How many different arrangements are there of the letters in:
A MAN, A PLAN, A CANAL: PANAMA

2. Eight rooks, 3 red, 2 white, and 3 blue, are to be placed on an 8-by-8 chessboard so that no two rooks are in the same rank or file (row or column). How many arrangements are possible?

3. Evaluate $\sum_{k=0}^{n} \frac{1}{k+1}\binom{n}{k}$ and $\sum_{k=0}^{n} \frac{(-1)^k}{k+1}\binom{n}{k}$

4. Fix positive integers N and k.

(a) How many integer solutions are there to:

$$x_1 + x_2 + \cdots + x_k = N, \quad x_i \geq 0 \quad ? \tag{1}$$

(*Hint*: The answer is a binomial coefficient.)

(b) Show there is a bijection between the set of integer solutions to:

$$x_1 + x_2 + \cdots + x_k \leq N, \quad x_i \geq 0 \tag{2}$$

and the set of integer solution to

$$x_1 + x_2 + \cdots + x_{k+1} = N, \quad x_i \geq 0\ . \tag{3}$$

Use this to find the number of integer solutions to the inequality (2).

(c) On the other hand, the number of integer solutions to (2) in part (b) is a sum of binomial coefficients. Use this to derive an identity for binomial coefficients.

5. (EXTRA) I invite one of three friends A, B, and C on 6 successive nights. How many arrangements are there such that no one friend is invited on three successive nights?