

Quiz Review – March 4, 1999

Here are some testable items for inclusion on future quizzes. We ought to have a quiz next week before Spring break, which should cover some part of this, to be negotiated today.

1. Definitions: Know *by rote, verbatim* the definitions of the following:
 - One-to-one function.
 - Onto function.
 - Coordinate function on a line.
 - “Betweenness” for points on a line.
 - Line segment.
 - Ray.
 - Angle.
 - Triangle.
2. Be able to prove:
 - $\sqrt[3]{5}$ is irrational, and similar statements.
 - Any statement from Section 1 of the class notes (Axiom list provided.)
3. Be able to form the negation of complex statements including quantifiers and conjunctions (and, or, if...then). Be able to form the contrapositive and converse of implications.

Here is a remark on the negation of complex sentences: The negation of a complex statement (one containing quantifiers or conjunctions) can be “simplified” step by step using our several rules for negating sentences, until the result contains only negations of simple statements. For example, a statement of the form “For all x , if $P(x)$ then $Q(x)$ and $R(x)$ ” has a negation which simplifies as follows:

$$\begin{aligned} \text{not(For all } x, \text{ if } P(x), \text{ then } Q(x) \text{ and } R(x)) &\equiv \\ \text{There exists } x \text{ such that not(if } P(x), \text{ then } Q(x) \text{ and } R(x)) &\equiv \\ \text{There exists } x \text{ such that } P(x) \text{ and not(} Q(x) \text{ and } R(x)) &\equiv \\ \text{There exists } x \text{ such that } P(x) \text{ and not}(Q(x) \text{) or not}(R(x) \text{) .} & \end{aligned}$$

Let’s consider a special case of a statement of this form: “For all real numbers x , if $x < 0$, then $x^3 < 0$ and $|x| = -x$.” Here we have $P(x) : x < 0$, $Q(x) : x^3 < 0$ and $R(x) : |x| = -x$. Therefore the negation of the statement is: “There exists a real number x such that $x < 0$, and $x^3 \geq 0$ or $|x| \neq -x$.”