

Assignment 3, Due Thursday Feb. 11

For the following exercises, use the existence and uniqueness of prime factorization of natural numbers, Stillwell, section 1.6

1. Show that $\sqrt{6}$ is irrational, that is, there are no natural numbers m, n such that $6n^2 = m^2$.
2. Show that $\sqrt{12}$ is irrational, that is, there are no natural numbers m, n such that $12n^2 = m^2$.
3. Make a conjecture about exactly which natural numbers have irrational square roots, in terms of the prime factorization of the natural number.
4. Prove your conjecture.
5. Show that $\sqrt[3]{3}$ is irrational, that is, there are no natural numbers m, n such that $3n^3 = m^3$.
6. Show that $\log_{10}(2)$ is irrational, that is, there are no natural numbers n and m such that $2 = 10^{m/n}$.