Homework for March 4, 1999

Here is a summary of rules for negating statements. Refer to the "appendices" which I handed out at the beginning of the term, and to section 4 of the class notes for more details.

- The negation of "A or B" is "not(A) and not(B)."
- The negation of "A and B" is "not(A) or not(B)."
- The negation of "For every x, P(x)" is "There exists x such that not(P(x))."
- The negation of "There exists an x such that P(x)" is "For every x, not(P(x))."
- The negation of "A implies B" is "A and not(B)." But note, many statements with implications have implicit universal quantifiers. For example, consider the statement: "If L and M are distinct lines with non-empty intersection, then the intersection of L and M consists of one point." This actually means: For every pair of lines L and M, if L and M are distinct and have non-empty intersection, then the intersection of L and M consists of one point." Therefore the negation uses both the rule for negation of sentences with universal quantifiers, and the rule for negation of implications: "There exists a pair of lines L and M such that L and M are distinct and have non-empty intersection, and the intersection does not consist of one point." This can be rephrased as: "There exists a pair of lines L and M such that L and M are distinct and have at least two points in their intersection."

1. Form the negation of each of the following sentences; no quantifier should appear within the scope of a not(), and the negation should be expressed in natural English.
   (a) Tonight I will go to a restaurant for dinner or to a movie.
   (b) Tonight I will go to a restaurant for dinner and to a movie.
   (c) If today is Tuesday, I have missed a deadline.
   (d) For all lines L, L has at least two points.
   (e) For every line L and every plane $\mathbb{P}$, if L is not a subset of $\mathbb{P}$, then $L \cap \mathbb{P}$ has at most one point.
   (f) (In the following statement, f is understood to be a function from a set A to a set B. The statement is the definition of f being onto.) For every $b \in B$ there exists an $a \in A$ such that $f(a) = b$.

(g) (Same context as the previous part. The statement is the definition of f being 1-to-1.) For every $a_1, a_2 \in A$ if $a_1 \neq a_2$, then $f(a_1) \neq f(a_2)$.

2. Same instructions as for the previous problem Watch out for implicit universal quantifiers.
   (a) If x is a real number, then $\sqrt{x^2} = |x|$.
   (b) If x is a natural number and x is not a perfect square, then $\sqrt{x}$ is irrational.
   (c) If n is a natural number, then there exists a natural number N such $N > n$.
   (d) If L and M are distinct lines, then either L and M do not intersect, or their intersection contains exactly one point.
3. Form the contrapositive of the following implications.
   (a) For every line L and every plane $\mathbb{P}$, if L is not a subset of $\mathbb{P}$, then $L \cap \mathbb{P}$ has at most one point.
   (b) For every line L and every plane $\mathbb{P}$, if L is not a subset of $\mathbb{P}$, then $L \cap \mathbb{P}$ has at most one point.
   (c) (In the following statement, f is understood to be a function from a set A to a set B. The statement is the definition of f being onto.) For every $b \in B$ there exists an $a \in A$ such that $f(a) = b$.
   (d) (Same context as the previous part. The statement is the definition of f being 1-to-1.) For every $a_1, a_2 \in A$ if $a_1 \neq a_2$, then $f(a_1) \neq f(a_2)$.
4. Form a/the converse of the implications in the previous exercise.
5. Prove Lemma 2.3 in the class notes.
6. Prove Theorem 3.8 in the class notes.