Mathematics 41 Midterm Exam I – F. Goodman
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Version 1

1. Solve the differential equation

$$(x^2 + y)\, dx + (x + e^y)\, dy = 0.$$

2. Solve the differential equation

$$(e^x + 1)\, dy = (y - ye^x)\, dx.$$

3. Solve the differential equation

$$y' = \frac{x^2 - 1}{y^2 + 1}.$$

4. **(a)** What does it mean for a function $\phi(t)$ to be a solution of a first order ordinary differential equation

$$y' = f(t, y)\,?$$

(b) What can you say about the existence and uniqueness of solutions to the linear first order differential equation

$$y' = f(t)\, y + g(t)$$

with initial condition $y(t_0) = y_0$?

5. Solve the differential equation

$$y' = \frac{y^3}{1 - 2xy^2}.$$

Little table of integrals:

$$\int \frac{1}{1 + e^x}\, dx = \int \frac{1 + e^x}{1 + e^x}\, dx - \int \frac{e^x}{1 + e^x}\, dx = x - \ln(1 + e^x) + c$$