

Math 34, First Midterm Review Questions
Fall, 2003

- (1) What is meant by a differential equation and what is meant by a solution to a differential equation? What is meant by an initial value problem? Illustrate your answers by an examples.
- (2) State precisely an existence and uniqueness theorem solutions to initial value problems. (Precisely means precisely: the hypotheses and conclusions must be accurately stated.)
- (3) Derive the general solution to the first order linear differential equation:

$$y' + p(t)y = g(t),$$

where p and g are continuous functions.

- (4) Derive the general solution to the logistic differential equation

$$y' = ry(1 - y/K),$$

where r and K are positive parameters. Identify the equilibrium points for this differential equation, and classify them as asymptotically stable or unstable. Make a rough sketch showing several solutions to the differential equation, and illustrating the typical behavior of the solutions.

- (5) For each of the following differential equations decide whether the equation is linear, separable, autonomous, exact (or none of these). If the equation is linear, separable, autonomous, or exact, solve the equation by an appropriate method.

(a) $y' + y^2 \sin(x) = 0$.

(b) $y' + 2ty = 2t$.

(c) $y' = \frac{\ln(ty)}{1 - t^2 - y^2}$.

(d) $y' = \frac{9x^2 + y - 1}{4y - x}$.

(e) $\frac{x}{(x^2 + y^2)^{3/2}} dx + \frac{y}{(x^2 + y^2)^{3/2}} dy = 0$.

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- (6) A tank initially holds 100 gallons of water and 50 oz. of salt. Water containing salt at a concentration of $.5(1 + .5\sin(t))$ oz/gal flows into the tank at a rate of 3 gal/minute, and the salt/water mixture flows out of the tank at the same rate. Find the amount of salt in the tank at any time.