Mathematics 25 Final Exam – F. Goodman
May 9, 1994

Scoring: The first problem counts for 30 points, the last for 20 points, all the others for 25 points. This exam paper has two pages and 8 problems.

1.

a) Give the definition of the derivative of a function.

b) State the Mean Value Theorem; don't forget to include the hypotheses.

c) State the Fundamental Theorem of Calculus; don't forget to include the hypotheses.

d) State the Chain Rule for the derivative of $f(g(x))$, where f and g are both differentiable functions.

e) Find the derivative of the function

$$F(x) = \int_1^{x^2} \sqrt{1+t}\, dt$$

HINT: $F(x) = f(x^2)$, where

$$f(x) = \int_1^{x} \sqrt{1+t}\, dt$$

2. Consider the function

$$f(x) = x^2 e^x,$$

whose first and second derivatives are

$$f'(x) = (x^2 + 2x)e^x \quad \text{and} \quad f''(x) = (x^2 + 4x + 2)e^x.$$

Graph the function using first and second derivative information and paying particular attention to local maxima and minima, points of inflection, asymptotes, and symmetry.

3. A Johnson County farmer with a fascination with elementary geometry has 400 ft. of fencing material. For reasons having little to do with agricultural economy, he decides to build two pig pens, one in the shape of circle and one in the shape of a square. Denote the length of fence devoted to the square pen by x, so the side of the square is $x/4$. What value of x maximizes the total area of the pens? What value of x minimizes the total area?

4. An airplane is flying at 1 mile altitude and at 500 mph, following a course which will take it directly over the house of the farmer in problem 3. (Meanwhile the farmer is sitting in his field with a pile of fencing, a calculus book, and a pad of paper.) Let s denote the distance between the plane and the house. At what rate is s decreasing when $s = 2$ miles?

5. Evaluate the following integrals. Remember to check any antiderivative by calculating a derivative!

a) $\displaystyle\int_1^4 x^2 + x + 1\, dx$

b) $\displaystyle\int_2^3 \frac{dx}{\sqrt{2x+1}}$

c) $\displaystyle\int \sin(x^2 + 2x)(x+1)dx$

d) $\displaystyle\int e^{x^2} x\, dx$

6. Find the area between the curves $y = x^2 - 6$ and $y = x$.

7. Calculate the volume of a sphere of radius R.

8. A hemispherical tank of radius 2 meters is filled with water. Calculate the work (in joules = newton-meters) required to pump the water to the top of the tank. Use the gravitational acceleration $g = 9.8$ m/sec^2 $= 9.8$ newton/kg and the density of water $\rho = 1000$ kg/m^3.