1. Max-Min problems

Note: I will draw pictures for the following exercises in lecture.

1. You are at point (A) on an island 1/2 mile from a straight shore. You need to get to a point (B) located on the shore, 5 miles from the point (C) on the shore closest to the island. Note that the triangle (ABC) is a right triangle with right angle at C and legs of length 1/2 and 5. You can swim 1 mi/hr and you can walk on the shore 3 mi/hr. What route should you take, swimming and walking, to get to point B in the least time. (To do this you will need to review the lecture given on Friday, March 14.)
2. A potter is making a coffee mug in the shap of a cylinder, with a bottom but no top. The mug is to hold 250 cm^3. What should be the radius and height of the cup to minimize the area, and thus the amount of clay used. (Presentation exercise.)
3. A box (with no top) is made out of a 6 by 6 inch square of sheet metal by first cutting small squares of side length s out of each corner, leaving a square with 4 rectangular tabs. The tabs are bent up and attached to one another, forming a box, with a square bottom, and with height x. How should x be chosen to maximize the volume of the box?

2. Newton's method

1. Read your notes from the lecture on Newton's method. Then put your notes away and derive the formula for the successive approximations in Newton's method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

   If you don't succeed, read your notes again, and then try to do it. Repeat this process until you are able to derive the formula without consulting your notes.
2. Start to calculate the 5th root of 7, "by hand" using Newton's method. 1 is too small and 2 is too large, so can use 1.5 as an initial guess, $x_0 = 1.5$. (Note that you have to solve the equation $f(x) = x^5 - 7 = 0$ by Newton's method.) Calculate x_1, x_2 and x_3 "by hand", that is using a pocket calculator rather than my Mathematica programs, or using Mathematica only as a fancy calculator. Write out the details of your computation carefully. (Presentation exercise.)
3. Calculate the 5th root of 7 "exactly" by using the "NaiveNewton" method from my demonstration notebook. Start with the same initial approximation $x_0 = 1.5$. (Presentation exercise.)
4. Find all of the solutions to $x^3 - 9x^2 + 2x + 3 = 0$. (Use Mathematica, and follow examples in the demonstration notebook. Presentation exercise.)
5. Find all solutions of $10251 - 3969x + 181x^2 + x^3 = 0$. (Use Mathematica, and follow examples in the demonstration notebook.)