**Homweork 3, Exercise 6.** Let $a$ be a positive number. Show by induction that for all natural numbers $n$,

$$(1+a)^n \geq 1+na.$$

**Solution:** Let $P(n)$ denote the statement

$$(1+a)^n \geq 1+na.$$

We need to show that $P(n)$ is valid for all natural numbers $n$. There are two steps to this. First we show that $P(1)$ is valid, and secondly we show that if for some $n$, $P(n)$ is valid, then also $P(n+1)$ is valid.

$P(1)$ is the statement

$$(1+a) \geq 1+a,$$

which is evidently true.

So now assume $P(n)$ is valid for some particular value of $n$. Then

$$\begin{aligned}(1+a)^{n+1} &= (1+a)^n(1+a)\\ &\geq (1+na)(1+a)\\ &= 1+na+a+na^2\\ &> 1+na+a\\ &= 1+(n+1)a,\end{aligned}$$

where the inequality in the second line results from the induction hypothesis and rules for inequalities, and the inequality in the fourth line just follows from $na^2 > 0$. This string of inequalities shows that $P(n+1)$ is also valid.

This completes the proof by induction.