Ú$_n$

The following is the generating function for exercise 31, page 250 in the text.

```
In[1]:= a = 1/((1 - x^2)(1 - x^5)(1 - x)(1 - x^7))
```

$$\text{Out[1]}= \frac{1}{(1-x)\,(1-x^2)\,(1-x^5)\,(1-x^7)}$$

The partial fraction expansion is *not* as I said in class, because the various explicit factors in the denominator are not relatively prime. Furthermore, the partial fraction expansion is not terribly useful for finding the power series:

```
In[2]:= Apart[a]
```

$$\text{Out[2]}= \frac{1}{70\,(-1+x)^4} - \frac{11}{140\,(-1+x)^3} + \frac{59}{280\,(-1+x)^2} - \frac{39}{112\,(-1+x)} + \frac{1}{16\,(1+x)} + \frac{x^2}{5\,(1+x+x^2+x^3+x^4)} + \frac{2+2x+3x^2+3x^3+2x^4+2x^5}{7\,(1+x+x^2+x^3+x^4+x^5+x^6)}$$

On the other hand, a symbolic mathematics program can easily compute the Taylor series up to any desired degree:

```
In[3]:= Series[a, {x, 0, 100}]
```

$$\begin{aligned}\text{Out[3]}= {} & 1 + x + 2x^2 + 2x^3 + 3x^4 + 4x^5 + 5x^6 + 7x^7 + 8x^8 + 10x^9 + 12x^{10} + 14x^{11} + \\ & 17x^{12} + 19x^{13} + 23x^{14} + 26x^{15} + 30x^{16} + 34x^{17} + 38x^{18} + 43x^{19} + 48x^{20} + \\ & 54x^{21} + 60x^{22} + 66x^{23} + 73x^{24} + 80x^{25} + 88x^{26} + 96x^{27} + 105x^{28} + \\ & 114x^{29} + 124x^{30} + 134x^{31} + 145x^{32} + 156x^{33} + 168x^{34} + 181x^{35} + \\ & 194x^{36} + 208x^{37} + 222x^{38} + 237x^{39} + 253x^{40} + 269x^{41} + 287x^{42} + \\ & 304x^{43} + 323x^{44} + 342x^{45} + 362x^{46} + 383x^{47} + 404x^{48} + 427x^{49} + \\ & 450x^{50} + 474x^{51} + 499x^{52} + 524x^{53} + 551x^{54} + 578x^{55} + 607x^{56} + \\ & 636x^{57} + 666x^{58} + 697x^{59} + 729x^{60} + 762x^{61} + 796x^{62} + 831x^{63} + \\ & 867x^{64} + 904x^{65} + 942x^{66} + 981x^{67} + 1021x^{68} + 1062x^{69} + 1105x^{70} + \\ & 1148x^{71} + 1193x^{72} + 1238x^{73} + 1285x^{74} + 1333x^{75} + 1382x^{76} + \\ & 1433x^{77} + 1484x^{78} + 1537x^{79} + 1591x^{80} + 1646x^{81} + 1703x^{82} + \\ & 1760x^{83} + 1820x^{84} + 1880x^{85} + 1942x^{86} + 2005x^{87} + 2069x^{88} + \\ & 2135x^{89} + 2202x^{90} + 2271x^{91} + 2341x^{92} + 2412x^{93} + 2485x^{94} + \\ & 2559x^{95} + 2635x^{96} + 2712x^{97} + 2791x^{98} + 2871x^{99} + 2953x^{100} + O[x]^{101}\end{aligned}$$